Signal Processing and Speech Communication Laboratory—Prof. G. Kubin
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Exam *Adaptive Systems* on 2016/5/20

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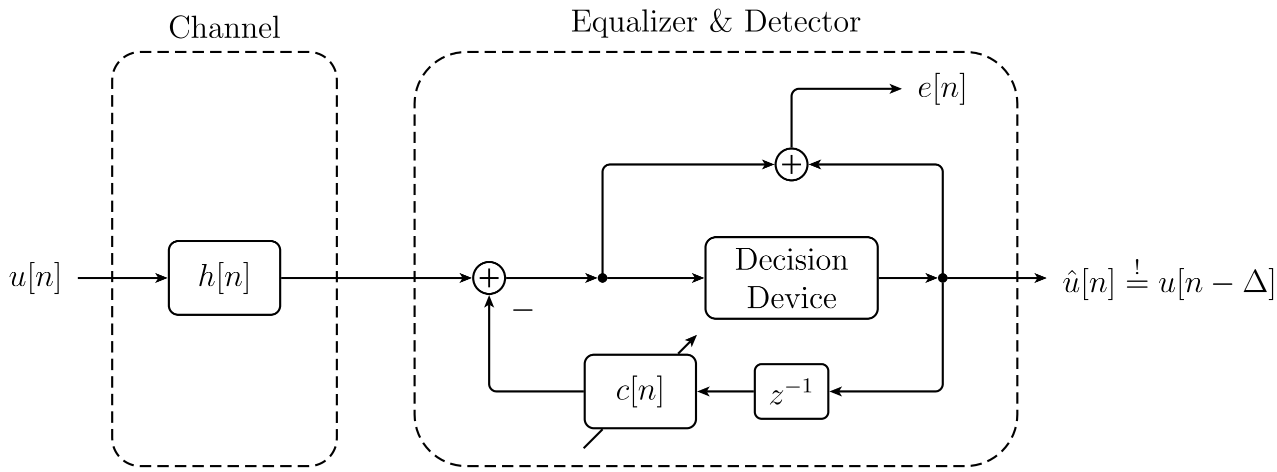
Exam duration: 3 hours
Maximum points: 100
Allowed material: mathematical formulary, calculator
You have to return this document after the exam.
Best wishes for a successful exam!

Problem 1 (33 Points)

Consider the given data transmission scenario over a channel with impulse response $h[n]$. We use a decision-feedback equalizer (feedback only) in decision-directed operation mode.



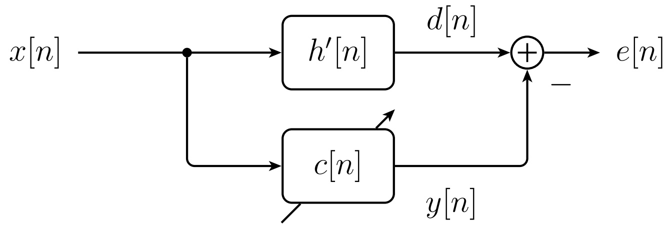
The symbols to be transmitted are $u[n] \in \{-1, 1\}$, which occur with the same probability. Consecutive symbols can be assumed to be uncorrelated. The decision device returns the sign of its input signal (zero is regarded as a positive number). The channel is given as

$$h[n] = 0.3\delta[n] + 1\delta[n-1] + 0.2\delta[n-2] - 0.8\delta[n-3].$$

(a) Consider a transmission over the given channel *without* an equalizer (decision device only). What is the ideal delay Δ that minimizes ISI (Inter Symbol Interference)? For this delay Δ, calculate the worst-case ISI and answer whether the channel's eye is open or not.

(b) Determine the sequence(s) for which the worst-case ISI occurs. Since all symbols occur with the same probability and since consecutive symbols are uncorrelated, all sequences are equally probable. An error occurs whenever the channel's eye is closed; calculate the error probability.

(c) Compare the decision-feedback equalizer to a simple system identification problem depicted below. Identify the signals $x[n]$, $y[n]$, and $d[n]$ in the schematic of the decision-feedback equalizer. If necessary, redraw the schematic to see the identities.



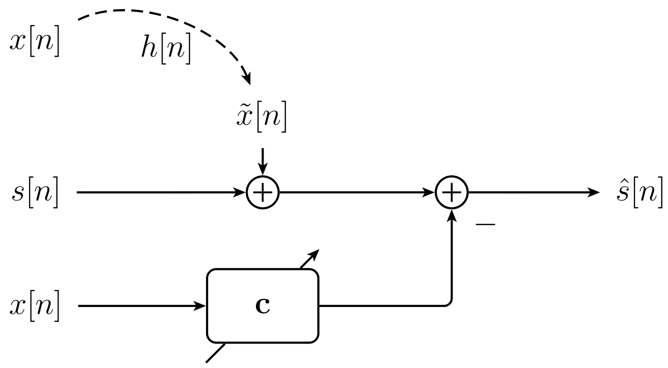
(d) Find the optimum solution $\mathbf{c}_{\text{opt}}$ for an N-coefficient ($N > 0$) decision-feedback equalizer in the sense of a minimum mean-squared error (MinMSE) for a general delay Δ and a general channel impulse response. **Hint:** Compute the cross-correlation vector $\mathbf{p}$ and the autocorrelation matrix $\mathbf{R}_{xx}$ and determine $\mathbf{c}_{\text{opt}}$ via the Wiener-Hopf equation.

(e) What is the best delay Δ for a decision-feedback equalizer with 2 coefficients $\mathbf{c} = [c_0, c_1]^T$ to minimize ISI? For this delay, give the optimum equalizer coefficients. Calculate the worst-case ISI and answer whether the equalizer can open the channel's eye or not.

(f) Finally, assume that only a decision-feedback equalizer with one coefficient is available, i.e., $\mathbf{c} = c_0$. Can this equalizer open the channel's eye? If not, how could you modify the schematic of the decision-feedback equalizer such that a single-coefficient equalizer suffices to open the channel's eye?

Problem 2 (34 Points)

Consider the following interference-cancelation problem:



A signal $s[n]$ (for simplicity, it is assumed that $s[n]$ is a white-noise signal with unit variance) is superimposed with a periodic interference $\tilde{x}[n]$, which is itself a filtered version of a periodic signal

$$x[n] = \cos\left(n\frac{\pi}{2} + \varphi\right)$$

where φ is uniformly distributed in $[0, 2\pi)$. It follows that the autocorrelation function of $x[n]$ is given as

$$r_{xx}[k] = \frac{1}{2}\cos\left(k\frac{\pi}{2}\right).$$

Assume further that the filter between $x[n]$ and $\tilde{x}[n]$ is a first-order IIR system with impulse response

$$h[n] = \left(\frac{1}{\sqrt{3}}\right)^n u[n].$$

You should now design an adaptive filter which has access to $x[n]$ in order to reconstruct $s[n]$ as accurately as possible, i.e., you should minimize

$$\mathbb{E}\left(e^2[n]\right) = \mathbb{E}\left((\hat{s}[n] - s[n])^2\right).$$

(a) What is the maximum filter length N you can choose for the adaptive filter $\mathbf{c}$? Justify your answer!

(b) Note that the signal $\tilde{x}[n]$ can be written as

$$\tilde{x}[n] = A\cos\left(n\frac{\pi}{2} + \varphi + \phi\right)$$

where A and ϕ are amplitude and phase changes introduced by the filter $h[n]$. Compute A and ϕ for the impulse response given above[1].

(c) For $N = 2$, determine the autocorrelation matrix $\mathbf{R}_{xx} = E\left\{\mathbf{x}[n]\mathbf{x}[n]^T\right\}$ of the signal $x[n]$ and the crosscorrelation vector $\mathbf{p} = E\left\{\tilde{x}[n]\mathbf{x}[n]\right\}$.

(d) Determine the Wiener solution $\mathbf{c}_{\text{opt}}$ for the first-order filter $\mathbf{c}$ ($N = 2$).

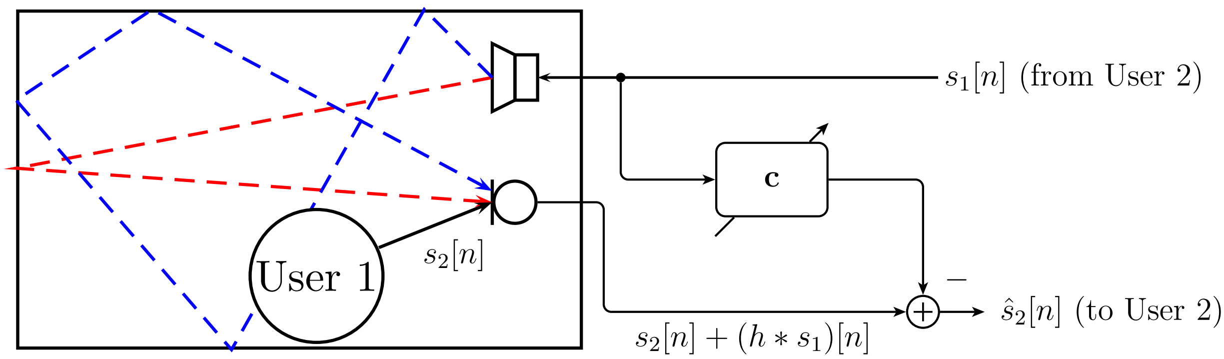
(e) Show that for the Wiener solution the mean-squared error vanishes, i.e., that $\mathbb{E}\left(e^2[n]\right) = 0$.

(f) It is interesting to observe that $\mathbb{E}\left(e^2[n]\right) = 0$ despite the fact that $\mathbf{c}_{\text{opt}}$ is a first-order FIR filter while $h[n]$ is a first-order IIR filter (infinite impulse response). How is this possible?

[1] If you cannot do it, you can use A and ϕ (instead of their values) in all following tasks.

Problem 3 (33 Points)

Imagine you are working at a speech technology company, and it is your task to design a voice-over-the-internet communication system (let's call it *Sqype*). The first problem you encounter is the following: Assume your users use a standard PC with an external speaker system and a hand-held microphone. It may occur that the signal from the speakers is captured by the microphone and transmitted to user on the other end of the line: the other user will hear herself with a slight delay, a very annoying phenomenon called *echo*. It is your task to design a digital filter which compensates that echo. In other words, you want to achieve $\hat{s}_2[n] = s_2[n]$.



We restrict ourselves to a first-order ($N = 2$) adaptive filter to keep the end-to-end delay small. Furthermore, we assume that the impulse response $h[n]$ (combining the effects of the microphone, the speaker, and the multipath propagation in the room) is given as

$$h[n] = \frac{13}{12}\delta[n] + \frac{5}{6}\delta[n-1] + \frac{1}{4}\delta[n-2].$$

(a) Assume that $s_1[n]$ and $s_2[n]$ are independent, zero-mean, white noise processes with variances σ_1^2 and σ_2^2. Derive the filter coefficients $\mathbf{c}_{\text{opt}}$ optimal in the MSE-sense, i.e., minimize

$$\mathbb{E}\left((s_2[n] - \hat{s}_2[n])^2\right).$$

(b) Under the assumption made above, compute the mean-squared error energy.
We are now looking at a more natural scenario: The signals $s_1[n]$ and $s_2[n]$ are still independent and zero-mean, but like speech signals we can assume that they have a low-pass character. In other words, for $i = 1, 2$,

$$s_i[n] = \frac{1}{\sqrt{2}}\left(w_i[n] + w_i[n-1]\right).$$

where $w_i[n]$ are now independent, white noise processes with variances σ_1^2 and σ_2^2.

(c) Verify that the variance of $s_i[n]$ is the same as the variance of $w_i[n]$.

(d) Compute the autocorrelation matrix $\mathbf{R}_{s_1 s_1}$.

(e) Derive again the filter coefficients $\mathbf{c}_{\text{opt}}$ optimal in the MSE-sense. What has changed compared to the case where $s_1[n]$ was white?

(f) Does the fact that $s_2[n]$ has a low-pass character influence the results in any way?

(g) Assuming finally that you compute the optimal filter coefficients iteratively, i.e., using either an LMS or a gradient search algorithm, under which signal conditions will the coefficients converge faster? In the white noise or in the low-pass case? Justify your answer!