Signal Processing and Speech Communication Laboratory
Graz University of Technology

Exam for

442.001 / SES.208UF Signal Processing
442.009 Fundamentals of Discrete-Time Signals and Systems

taking place on 13.05.2026

Duration: 180 minutes, Achievable points: 100

Permitted aids: provided SPSC formulary, scientific (*non*-alphanumeric) calculator, supplied (*no* personally brought) paper, drinks and snacks.

! Important !

The problem statement and the formulary must be returned at the end of the exam!

Write your name and matriculation number on all sheets of your solution!

All results must be explained in a comprehensible way!

Problem 1 (25 Points)

Given are the following discrete-time signals:

$$x[n] = \cos\left(\frac{\pi}{2}n\right) \qquad\qquad h[n] = \begin{cases} 0 & n < 0 \\ 1 & 0 \leq n < 2 \\ 0 & 2 \leq n \end{cases}$$

(a) Are these sequences periodic signals? If yes, what is the period length N_0?

(b) Assume that the signals were sampled with a sampling frequency of $f_s = 800$ Hz and the Nyquist criterion was obeyed. What period T_0 in seconds would correspond to the discrete period N_0?

(c) Let $\hat{x}[n] = \begin{cases} x[n] & \text{if} \quad 0 \leq n \leq 2 \\ 0 & \text{otherwise.} \end{cases}$

Compute analytically the linear convolution $y[n] = (\hat{x} * h)[n]$ and sketch the signals $\hat{x}[n]$, $h[n]$ and $y[n]$. What is the length N_y of the result of a linear convolution of two signals with length N_x and N_h in general?

(d) Use the formulary to compute the multiplication of the signals $z[n] = x[n] \cdot h[n]$ using the DTFT, i.e., compute $z[n] = \text{DTFT}^{-1}\{\text{DTFT}\{x[n] \cdot h[n]\}\}$.

Problem 2 (25 Points)

The basis $\mathcal{B}$ is spanned by $2K$ base signals $b_k[n] \in \mathbb{C}$ of the form

$$b_k[n] = e^{j\frac{\pi}{4}kn}$$

(with $k = -K, ..., 0, 1, \ldots, K-1$ and $n = -\frac{N}{2}, -\frac{N}{2}+1, \ldots, \frac{N}{2}-2, \frac{N}{2}-1$). Let $K = 3$ and $N = 8$.

(a) Determine whether the basis $\mathcal{B}$ is orthonormal. Justify your answer (mathematically or by explanation)!
Hint: Do all combinations behave differently, or is there a pattern that allows generalization?

(b) A signal $x_1[n] = A + \frac{A}{2}cos(\frac{\pi}{4}n)$ with $n = -\frac{N}{2}, -\frac{N}{2}+1, \ldots, \frac{N}{2}-2, \frac{N}{2}-1$ is to be represented as the sum of the $2K$ basis signals $\{b_k[n]\}_{k=-K}^{K-1}$

$$x_1[n] = \sum_{k=-K}^{K-1} \alpha_k \cdot b_k[n].$$

Calculate the coefficients $\alpha_k = \langle x_1, b_k \rangle$ and plot them over the corresponding angular frequency $\omega = \frac{\pi}{4}k$.

(c) Is it possible to represent a signal $x_2[n] = \frac{A}{2}\sin(\frac{\pi}{4}n)$ with $n = -\frac{N}{2}, -\frac{N}{2}+1, \ldots, \frac{N}{2}-2, \frac{N}{2}-1$ as the sum of the $2K$ basis signals $\{b_k[n]\}_{k=-K}^{K-1}$ with $\boldsymbol{\beta_k \in \mathbf{R}}$

$$x_2[n] = \sum_{k=-K}^{K-1} \beta_k \cdot b_k[n].$$

Justify your answer (mathematical derivation or verbal argument)!

(d) The signal $x_1[n]$ is modulated with $h[n] = e^{j\frac{\pi}{4}n}$, the resulting signal $y[n]$ equals

$$y[n] = x_1[n]h[n].$$

Define new base signals $c_k[n] \in \mathbb{C}$ for a new basis $\mathcal{C}$ (with $k = -K, 1, \ldots, K-1$ and $n = -\frac{N}{2}, -\frac{N}{2}+1, \ldots, \frac{N}{2}-2, \frac{N}{2}-1$ with $K = 3$ and $N = 8$) so that

$$y[n] = \sum_{k=-K}^{K-1} \alpha_k \cdot c_k[n].$$

Problem 3 (25 Points)

(a) Consider the following two systems:

- $y_1[n] = x_1[n] - x_1[n-1]$
- $Y_2(z) = (1 - z^{-1})X_2(z)$

Investigate whether these systems are linear and time-invariant (LTI), and determine whether they are BIBO stable.

(b) Choose one of the two systems from (a) that is a LTI system $H_L(z)$. Determine its impulse response in the time domain and sketch the magnitude response of the system and answer the following questions:

- What type of behavior do you observe (low-pass, high-pass, band-pass, or notch)?
- Does the system have a finite or infinite impulse response?
- Is the system causal, non-causal, or anti-causal?

(c) For the system $H_L(z)$ from task (b), determine a scaling factor k such that the normalized system

$$H_N(z) = k \cdot H_L(z)$$

satisfies

$$\max_{\theta} \left| H_N(e^{j\theta}) \right| = 1.$$

(d) Determine the inverse system of (c), i.e., $H_N^{-1}(z)$ (if you have not solved (c), use $k = 1$). Discuss:

- What type of behavior do you observe (low-pass, high-pass, band-pass, or notch)?
- Is the system BIBO stable?

(e) Consider a modified system with impulse response

$$h_A[n] = h_L[n] * \delta[n+1].$$

Determine the impulse response in both the time domain and the Z-domain. Discuss what changes with respect to magnitude response, phase response, causality, and BIBO stability.

Problem 4 (25 Points)

You are given a system that observes a *continuous-time signal* $x(t)$ and stores its samples in a discrete-time memory sequence $x[n]$. The discrete-time index is defined over the finite range

$$n \in \{0, 1, \ldots, L-1\}.$$

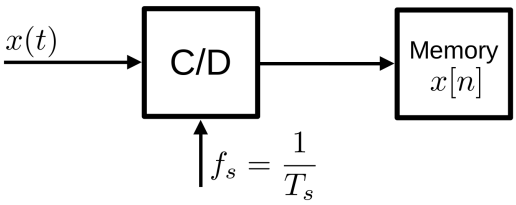


Abbildung 1: Block diagram of the sampling and memory system

(a) Consider the complex exponential basis signals

$$b_k[n] = e^{j\theta_k n}, \qquad \theta_k = \frac{2\pi}{L}k.$$

Determine how many such basis signals are required to span the signal space $\mathcal{B}$ that can represent *any signal stored in the memory* $x[n]$ for $L = 4$. Briefly justify your answer.

(b) Let the continuous-time signal be given by

$$x(t) = A\cos(2\pi 100t),$$

with sampling frequency $f_s = 400Hz$.

1. Compute the sampled sequence $x[n]$ and simplify the result as far as possible.
2. Compute the DFT $X[k]$ of $x[n]$ for length $L = 4$
3. Sketch $|X[k]|$ over θ_k.

(c) Now assume that the memory is *infinite*, i. e. $n \in \mathbb{Z}$, but only $L = 4$ values are nonzero such that

$$\hat{x}[n] = (u[n] - u[n-L])\, x[n].$$

1. Compute the DTFT $\hat{X}(e^{j\theta})$ of $\hat{x}[n]$ make use of the transform pairs/identities from the formulary.
2. Sketch the magnitude spectrum $|\hat{X}(e^{j\theta})|$ for $\theta \in [-\pi, \pi]$.

(d) Compare your results from tasks (b) and (c). Discuss whether the DFT samples the DTFT for the given case. Support your argument using Parseval's relations:

$$\sum_{m=0}^{M-1} |x[m]|^2 = \frac{1}{M} \sum_{k=0}^{M-1} |X[k]|^2,$$

$$\sum_{m=-\infty}^{\infty} |x[m]|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\Theta})|^2 \, d\Theta.$$

(e) Let

$$x_2(t) = A \cos(2\pi \cdot 100\, t),$$

with sampling frequency $f_s = 400\,\text{Hz}$ and $L = 5$.

- Explain which effect occurs and why.
- Discuss whether your conclusions from task (d) still hold in this case.