Signal Processing and Speech Communication Laboratory
Graz University of Technology

Exam for

442.001 / SES.208UF Signal Processing
442.009 Fundamentals of Discrete-time Signals and Systems

taking place on 06.03.2024

Duration: 180 minutes, Achievable points: 100

Permitted aids: provided SPSC formulary, scientific (*non*-alphanumeric) calculator, supplied (*no* personally brought) paper, drinks and snacks.

! Important !

The problem statement and the formulary must be returned at the end of the exam!

Write your name and matriculation number on all sheets of your solution!

Problem 1 (25 Points)

A signal $x[n]$ where $0 \leq n < N$ should be represented as a combination of K base signals $\{b_k[n]\}_{k=0}^{K-1}$ where $0 \leq n < M$ with corresponding coefficients $\{a_k\}_{k=0}^{K-1}$ as

$$x[n] = \sum_{k=0}^{K-1} a_k \cdot b_k[n].$$

(a) In general, how would you choose the length M of the base signals? Discuss the cases (i) $M < N$, (ii) $M = N$ and (iii) $M > N$.

(b) How many base signals K are certainly sufficient to represent an arbitrary signal

(i) $x \in \mathbb{R}^N$ with $b_k \in \mathbb{R}^M$,

(ii) $x \in \mathbb{C}^N$ with $b_k \in \mathbb{C}^M$,

for $k \in \{0 \ldots K-1\}$ and **real-valued coefficients** $a_k \in \mathbb{R}$ with suitably chosen M? Using a simple example for $N = 2$, illustrate the case that a signal space cannot be sufficiently spanned with too few base signals.

(c) How can you compute the coefficients a_k for a general signal x and orthogonal base signals b_k?

(d) Let $x[n]$ be given as

$$x[n] = \begin{cases} A & \text{if} \quad n < N_0 \\ 0 & \text{otherwise} \end{cases}$$

where $0 < N_0 \leq N$ and $b_k[n] = \delta[n-k]$. Determine the coefficients a_k.

(e) Which criteria must an orthonormal basis obey? Is the basis in Task (d) an orthonormal basis? Prove!

Problem 2 (25 Points)

Given are the following discrete-time signals:

$$x[n] = \cos\left(\frac{\pi}{2}n\right) \qquad\qquad h[n] = \begin{cases} 0 & n < 0 \\ 1 & 0 \leq n < 2 \\ 0 & 2 \leq n \end{cases}$$

(a) Are these sequences periodic signals? If yes, what is the periode length N_0?

(b) Assume that the signals were sampled with a sampling frequency of $f_s = 800$ Hz und the Nyquist criterion was obeyed. What period T_0 in s would correspond to the discrete period N_0?

(c) Let $\hat{x}[n] = \begin{cases} x[n] & \text{if} \quad 0 \leq n \leq 2 \\ 0 & \text{otherwise.} \end{cases}$

Compute analytically the linear convolution $y[n] = (\hat{x} * h)[n]$ and sketch the signals $\hat{x}[n]$, $h[n]$ und $y[n]$. What is the length N_y of the result of a linear convolution of two signals with length N_x and N_h in general?

(d) Use the formulary to compute the multiplication of the signals $z[n] = x[n] \cdot h[n]$ using the DTFT, i.e., compute $z[n] = \text{DTFT}^{-1}\{\text{DTFT}\{x[n] \cdot h[n]\}\}$.

Problem 3 (25 Points)

Consider a causal, discrete-time LTI-system with system function $H(z)$. Its poles are given as $z_{\infty,1} = -0.75$ and $z_{\infty,2} = 0.5$, its zeros are given as $z_{0,1} = 1.5j$ and $z_{0,2} = -1.5j$.

(a) Sketch the pole-zero diagram and determine the region of convergence (ROC). Is the system BIBO stable? Explain!

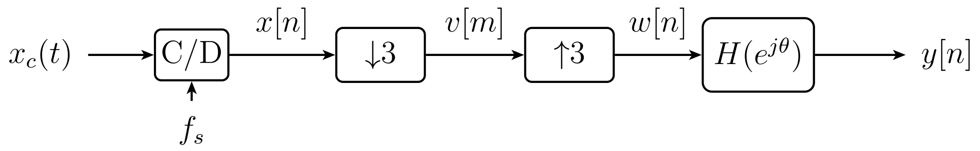
(b) A system is minimum-phase if it has a causal and stable inverse. Is $H(z)$ minimum-phase? Explain!

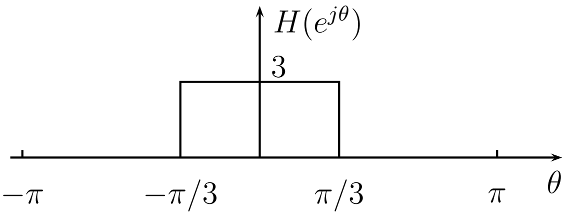
(c) Assume that the maximum of the magnitude response satisfies $\max_\theta |H(e^{j\theta})| = 1$. Determine the system function $H(z)$.

(d) Determine the impulse response $h[n]$ of the system. Is $h[n]$ absolut summable, i.e., $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$? Explain!

Problem 4 (25 Points)

Consider the following multirate system with an ideal lowpass filter $H(e^{j\theta})$:





The continuous-time signal $x_c(t) = \cos(2\pi f_0 t)$ with frequency $f_0 = 200\,\text{Hz}$ is sampled at a rate $f_s = 0.8\,\text{kHz}$ in order to obtain the discrete-time signal $x[n]$.

(a) Determine the discrete-time signal $x[n]$. Can you find an alternative choice for f_0 yielding the same discrete-time signal $x[n]$?

(b) Sketch the signals $x[n]$, $v[m]$ und $w[n]$ for the input signal $x_c(t)$. Sketch at least two perods of the signals.

(c) Sketch the Fourier transforms of the signals $x[n]$, $v[m]$ und $w[n]$ for the input signal $x_c(t)$ and mind the correct amplitude scaling. Sketch at least two perods of the spectra.

(d) Does $y[n] = x[n]$ hold? If not, sketch the frequency response of an ideal filter $H(e^{j\theta})$ such that $y[n] = x[n]$ holds.

(e) Which requirement must f_0 fulfill for a given sampling frequency such that $y[n] = x[n]$ holds?

(f) Can information loss occur in this multirate system? Explain!