Signal Processing and Speech Communication Laboratory
Graz University of Technology

Exam for

442.001 / SES.208UF Signal Processing
442.009 Fundamentals of Discrete-time Signals and Systems

taking place on 24.09.2025

Duration: 180 minutes, Achievable points: 100

Permitted aids: provided SPSC formulary, scientific (*non*-alphanumeric) calculator, supplied (*no* personally brought) paper, drinks and snacks.

! Important !

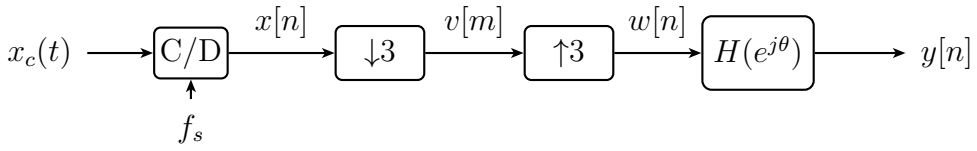
The problem statement and the formulary must be returned at the end of the exam!

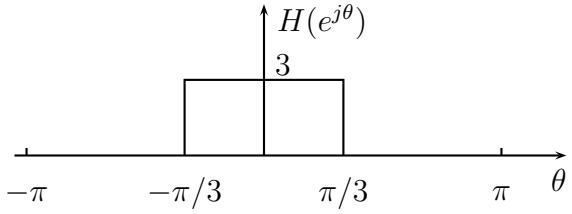
Write your name and matriculation number on all sheets of your solution!

All results must be explained in a comprehensible way!

Problem 1 (25 Points)

Consider the following multirate system with an ideal lowpass filter $H(e^{j\theta})$:





The continuous-time signal $x_c(t) = \cos(2\pi f_0 t)$ with frequency $f_0 = 200\,\text{Hz}$ is sampled at a rate $f_s = 0.8\,\text{kHz}$ in order to obtain the discrete-time signal $x[n]$.

(a) Determine the discrete-time signal $x[n]$. Can you find an alternative choice for positive f_0 yielding the same discrete-time signal $x[n]$?

(b) Sketch the signals $x[n]$, $v[m]$ und $w[n]$ for the input signal $x_c(t)$. Sketch at least two periods of the signals.

(c) Sketch the Fourier transforms of the signals $x[n]$, $v[m]$ und $w[n]$ for the input signal $x_c(t)$ and mind the correct amplitude scaling. Sketch at least two periods of the spectra.

(d) Does $y[n] = x[n]$ hold? If not, sketch the frequency response of an ideal filter $H_I(e^{j\theta})$ such that $y[n] = x[n]$ holds.

(e) Which requirement must f_0 fulfill with the filter $H(e^{j\theta})$ for a given sampling frequency such that $y[n] = x[n]$ holds?

Problem 2 (25 Points)

The base $\mathcal{B}$ is spanned by $2K$ base signals $b_k[n] \in \mathbb{C}$ of the form

$$b_k[n] = \frac{1}{N} e^{j\frac{\pi}{4}kn}$$

(with $k = -K, ..., 0, 1, \ldots, K-1$ and $n = -\frac{N}{2}, -\frac{N}{2}+1, \ldots, \frac{N}{2}-2, \frac{N}{2}-1$). Let $K = 3$ and $N = 8$.

(a) Determine whether the base $\mathcal{B}$ is orthonormal.

(b) A signal $x_1[n] = A + \frac{A}{2} cos(\frac{\pi}{4}n)$ with $n = -\frac{N}{2}, -\frac{N}{2}+1, \ldots, \frac{N}{2}-2, \frac{N}{2}-1$ is to be represented as the sum of the $2K$ basis signals $\{b_k[n]\}_{k=-K}^{K-1}$

$$x_1[n] = \sum_{k=-K}^{K-1} \alpha_k \cdot b_k[n].$$

Calculate the coefficients $\alpha_k = \langle x_1, b_k \rangle$ and plot them over the corresponding angular frequency $\omega = \frac{\pi}{4}k$.

(c) Is it possible to represent a signal $x_2[n] = \frac{A}{2} sin(\frac{\pi}{4}n)$ with $n = -\frac{N}{2}, -\frac{N}{2}+1, \ldots, \frac{N}{2}-2, \frac{N}{2}-1$ as the sum of the $2K$ basis signals $\{b_k[n]\}_{k=-K}^{K-1}$ with $\boldsymbol{\beta_k \in \mathbf{R}}$

$$x_2[n] = \sum_{k=-K}^{K-1} \beta_k \cdot b_k[n].$$

Justify your answer (mathematical derivation or sentences)!

(d) The signal $x_1[n]$ is modulated with $h[n] = e^{j\frac{\pi}{4}n}$, the resulting signal $y[n]$ equals

$$y[n] = x_1[n]h[n].$$

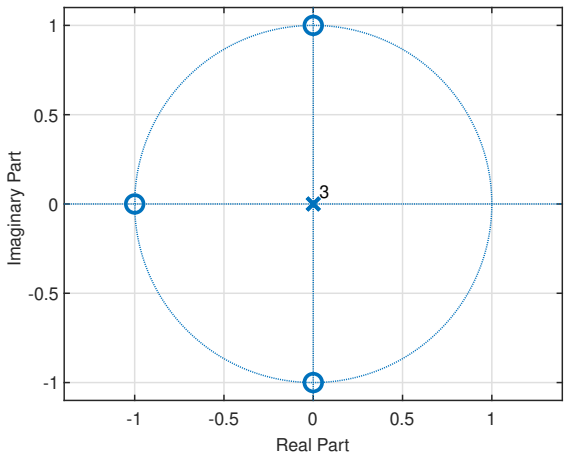
Define new base signals $c_k[n] \in \mathbb{C}$ for a new base $\mathcal{C}$ (with $k = -K, 1, \ldots, K-1$ and $n = -\frac{N}{2}, -\frac{N}{2}+1, \ldots, \frac{N}{2}-2, \frac{N}{2}-1$ with $K = 3$ and $N = 8$) so that

$$y[n] = \sum_{k=-K}^{K-1} \alpha_k \cdot c_k[n].$$

(e) Can the base $\mathcal{C}$ represent a signal $s[n] = cos(-\frac{3}{4}\pi n)$? Justify your answer!

Problem 3 (25 Points)

A given system H has the following pole zero plot.



(a) Determine the transfer function $H(z)$ and simplify it as much as possible.

(b) Sketch the magnitude response $|H(e^{j\theta})|$ of the system. Which type of filter is this system (lowpass, highpass, bandpass, or notch)?

(c) Compute $max_\theta(|H(e^{j\theta})|)$ and find a normalization gain k such that $max_\theta(|H(e^{j\theta})|) = 1$.

(d) Determine the impulse response $h[n]$ of the system. Is the system causal and BIBO stable? Justify your answer.

(e) Decide whether the system is an FIR (finite impulse response) or IIR (infinite impulse response) filter. Sketch the phase response of the filter in the interval $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$.

Problem 4 (25 Points)

Consider the system in the block diagram below. The ADC and DAC operate at sampling frequency f_s. The continuous-time input signal lies in the finite signal space $\mathcal{B} = \text{span}\{b_m(t)\}_{m=0}^{2}$, $b_m(t) = e^{j2\pi m f_0 t}$ and has coefficient vector $\alpha = [0, 1, 1]$, i.e.

$$x(t) = \sum_{m=0}^{2} \alpha_m e^{j2\pi m f_0 t}$$

with the sampling frequency satisfying $2f_0 < \frac{f_s}{2}$. The discrete-time LTI system H has impulse response given by

$$H(z) = \tfrac{1}{2}\big(1 - (1+\sqrt{2})z^{-1} + (1+\sqrt{2})z^{-2} - z^{-3}\big).$$

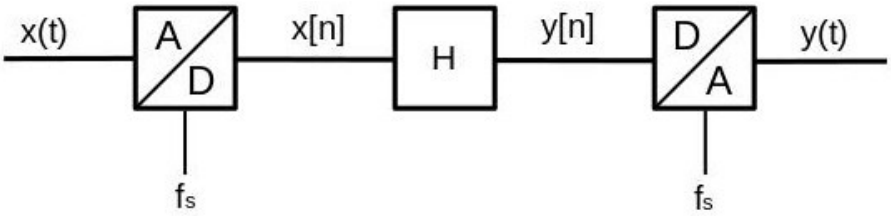


Abbildung 1: block diagram

(a) Determine the discrete-time signal $x[n]$ (sampled at f_s).

(b) Compute and sketch the DTFT $X(e^{j\theta})$ over $\theta \in [-\pi, \pi]$ (equivalently $f \in [-f_s/2, f_s/2]$): magnitude and phase.

(c) For $H(z)$:

1. Sketch the pole zero plot.
2. Indicate where the DTFT magnitude $|H(e^{j\theta})|$ is obtained on that plot
3. Classify the filter (lowpass/highpass/bandpass/bandstop) and justify
4. Determine if the system H is BIBO-stable

(d) For $f_0 = \dfrac{f_s}{8}$, mark the angular frequencies $\theta_1 = 2\pi\dfrac{f_0}{f_s}$ and $\theta_2 = 2\pi\dfrac{2f_0}{f_s}$ in the pole zero plot.

(e) For $f_0 = \dfrac{f_s}{8}$ compute and sketch the magnitude spectrum of $y[n]$. Is it necessary to use the same signal space $\mathcal{B}$ for $y[n]$? Justify.
Hint: Use your solution of task (c) and (d)