Signal Processing and Speech Communication Laboratory
Graz University of Technology

Exam for

442.001 / SES.208UF Signal Processing
442.009 Fundamentals of Discrete-Time Signals and Systems

taking place on 21.01.2026

Duration: 180 minutes, Achievable points: 100

Permitted aids: provided SPSC formulary, scientific (*non*-alphanumeric) calculator, supplied (*no* personally brought) paper, drinks and snacks.

! Important !

The problem statement and the formulary must be returned at the end of the exam!

Write your name and matriculation number on all sheets of your solution!

All results must be explained in a comprehensible way!

Problem 1 (25 Points)

(a) Consider the discrete-time signal $s[n] = \cos\left(\frac{3\pi^2}{7}n\right)$. Determine whether $s[n]$ is periodic. If it is periodic, find its fundamental period. Provide proof in case it is not periodic.

(b) Consider the discrete-time system $y[n] = n + x[n]$. Prove or disprove that this system is linear (i.e., additive and homogeneous) and/or time-invariant.

(c) Consider the discrete-time system $y_[n] = 3x[n] - x[n-1]$. Prove that this system is linear (i.e., additive and homogeneous) and time-invariant.

(d) For the system in Task (c) determine

- the *impulse response* $h[n]$, and
- the *step response* $s[n]$.

(e) Let $w[n] = \delta[n] + 2\delta[n-1]$. Explicitly compute the *linear convolution* $y[n] = (w * h)[n]$ and sketch the system output $y[n]$.

Problem 2 (25 Points)

Consider the discrete-time signal $x[n] = \cos(\theta_0 n) \cdot w[n]$, where the rectangular window of length N is defined as

$$w[n] = \begin{cases} 1 & \text{if} \quad 0 \leq n < N, \\ 0 & \text{otherwise.} \end{cases}$$

Thus, the observed signal is a finite-length segment of a periodic cosine. Let $\theta_0 = \frac{\pi}{2}$ and $N = 4$.

(a) Compute the DTFT $X(e^{j\theta})$ of the windowed cosine. You may use the formulary. Sketch the magnitude $|X(e^{j\theta})|$ for $\theta \in [-\pi, \pi]$.

(b) Compute the 4-point DFT $X[k]$ of $x[n]$ for $0 \leq n < 4$. You may use the formulary. Sketch the magnitude $|X[k]|$.

(c) Compute the 5-point DFT of the zero-padded signal

$$\tilde{x}[n] = \begin{cases} x[n] & \text{if} \quad n < 4, \\ 0 & \text{otherwise,} \end{cases}$$

for $0 \leq n < 5$ *without* using the formulary. Thus, the window length remains $N = 4$, but the DFT size is now 5. Sketch the magnitude $|\tilde{X}[k']|$. Hint:

$$\sum_{n=0}^{N-1} q^n = \frac{1 - q^N}{1 - q},$$

(d) Sketch now the DTFT $|X(e^{j\theta})|$ and the DFTs $|X[k]|$ and $|\tilde{X}[k]|$ for $\theta \in [-\pi, \pi]$ in the same plot, i.e., you need to map the DFT indices k' to their corresponding frequencies θ.

(e) Explicitly state the basis functions of the DTFT and DFT transforms you computed above. Explain conceptually how the DFT and DTFT are related in terms of their respective signal spaces and basis functions. In your explanation, you may refer to the unit circle.

Problem 3 (25 Points)

Consider a system with impulse response

$$h[n] = \left(2^{n-1} + (-2)^{n-1}\right) \cdot u[-n+1]$$

(a) Sketch the impulse response of this system for $-5 \leq n < 5$. Is the system causal, acausal or anti-causal? Explain.

(b) Compute the Z-transform $H(z)$ of the impulse response *without* the use of transform pairs/identities from the formulary. Hint: $\sum_{n=0}^{\infty} q^n = \frac{1}{1-q}$ for $|q| < 1$.

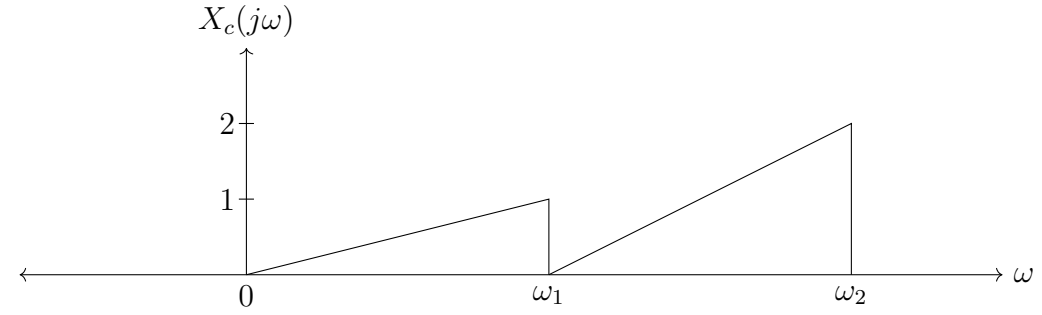
(c) Determine the poles/zeros and the region of convergence of $H(z)$. Hint: your derivations from Task (b) may be insightful. Sketch the pole-zero plot. Is the system stable? Explain.

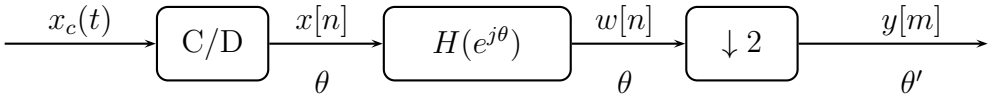
(d) Given your pole/zero plot, determine the frequencies θ where $|H(e^{j\theta})|$ has its maxima and minima. Explain. In case you could not solve Task (b) and Task (c), assume that $H(z)$ has poles at $z_\infty \in \{-2, 0, 2\}$.

(e) Determine further the specific values $\max_\theta |H(e^{j\theta})|$ and $\min_\theta |H(e^{j\theta})|$, and sketch $|H(e^{j\theta})|$ for $\theta \in [-\pi, \pi]$.

Problem 4 (25 Points)

A complex-valued continuous-time signal $x_c(t)$ has the Fourier transform depicted below and is run trough the given processing chain.





Let $\omega_1 = 3s^{-1}$, $\omega_2 = 8s^{-1}$, and let $x[n]$ denote the signal obtained by sampling $x_c(t)$ with sampling frequency f_s.

(a) Determine the smallest sampling frequency f_s as a function of ω_1 and/or ω_2, such that no *aliasing* occurs during the sampling process. For this sampling frequency and the concrete values for ω_1 and ω_2, sketch the DTFT $X(e^{j\theta})$ of the sampled sequence $x[n]$ for $\theta \in [-2\pi, 2\pi]$.

(b) Determine the smallest sampling frequency f_s as a function of ω_1 and/or ω_2, such that no *information loss* occurs during the sampling process. Explain the difference to Task (a). For this sampling frequency and the concrete values for ω_1 and ω_2, sketch the DTFT $X(e^{j\theta})$ of the sampled sequence $x[n]$ for $\theta \in [-2\pi, 2\pi]$.

(c) Assume the continuous-time signal was sampled at a sampling frequency $f_s = \frac{4}{\pi}$Hz. Design an idealized filter $H(e^{j\theta})$ that extracts only the spectral portion of the continuous-time signal in the interval $0 \leq \omega \leq \omega_1$. State the filter's cut-off frequencies and sketch the DTFTs $H(e^{j\theta})$ and $W(e^{j\theta})$ of the filter resp. the filtered sequence.

(d) The filtered sequence $w[n]$ is now *downsampled* to obtain the signal $y[m]$. Sketch the DTFT $Y(e^{j\theta'})$ of $y[m]$ for $\theta' \in [-2\pi, 2\pi]$. Can the frequency components in the band $0 \leq \omega \leq \omega_1$ still be recovered without information loss? Explain.