Loudspeaker Simulation considering Suspension Creep

Audio Engineering Project

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1 Introduction

Loudspeaker models are getting more complex and try to model more occurring physical effects. One of these effects is viscoelastic creep which affects polymers of the loudspeaker suspension. The excursion of the membrane gets time and frequency dependent and an increase of the excursion at low frequencies occurs. This can lead to problems especially in microspeakers due to place limitations and the lack of a spider. Models considering creep usually work in frequency domain and therefore are restricted to linearity. To be able to consider nonlinearities like the force-factor BL we make our calculations in time domain.

We start with a traditional loudspeaker model and replace the spring with a Standard Linear Solid model to describe the viscoelastic creep. Then we send stepped sine excitations into the system, calculate the differential equation and solve it numerically with the MATLAB ode-solver. After calculating the maximum excitation in steady state we can observe the increase at low frequencies. This concept can be improved and expanded upon further.

2 Traditional loudspeaker model

The traditional loudspeaker model is a linear model which represents the main properties of a loudspeaker. The most common type of loudspeakers is the electrodynamic loudspeaker. Its goal is to produce a sound signal from an electrical signal. You can split the model into three parts which represent the properties of the loudspeaker in the electrical, the mechanical and the acoustic domain. Usually you can merge the distinct parts into a single model, completely represented in the electrical domain. This brings the advantage that you can use well known methods of electrical circuit theory to analyse this model. The traditional loudspeaker model is represented by a set of parameters which are known as the Thiele-Small-parameters. These parameters describe the model in a way that the parameters represent real physical values like the mass of the membrane or the electrical impedance of the voice-coil.

2.1 Electrodynamic loudspeaker

The electrodynamic loudspeaker is the most common type of loudspeakers. It is an electroacoustic transducer which transforms an electrical signal into a sound signal.

A typical electrodynamic loudspeaker consists of a membrane which is fixed on the basket over the suspension. The voice-coil is attached to the membrane and is arranged in a gap between a permanent magnet. There the voice-coil can move in an axial way. The electrical signal is induced into the voice-coil, which creates a magnetic field that interacts with the magnetic field of the permanent magnet. This produces a force which moves the voice-coil and the attached membrane to reproduce sound out of the electrical signal.

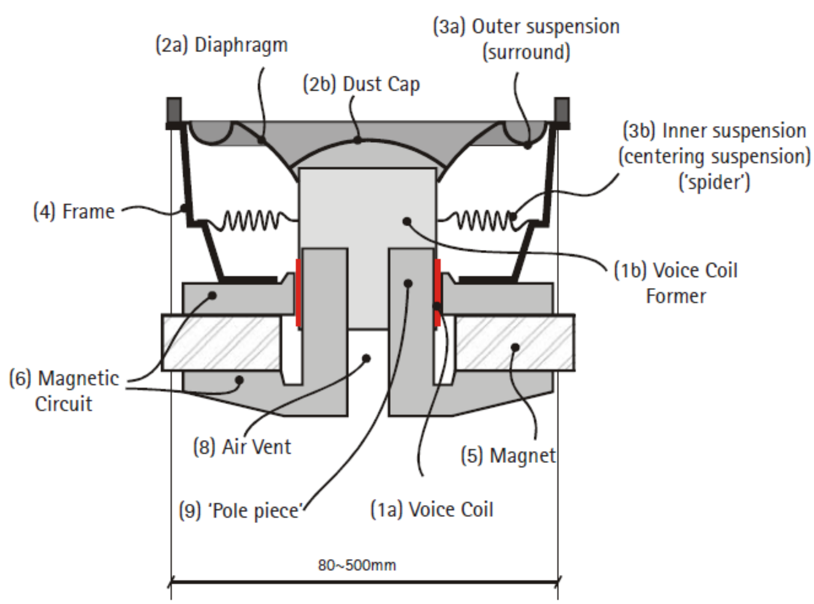


Figure 1: Basic components of an electrodynamic loudspeaker. [5]

In big loudspeakers there are normally two suspensions. One, the surround, which links the membrane to the basket and a second, the spider, which constrains the voice-coil to move only in one axis. The membrane, also called diaphragm, is usually made out of paper or plastic materials and has the shape of a cone. The design of a loudspeaker is traditionally circular. In figure 1 you can see a typical structure of an electrodynamic loudspeaker.

Smaller variants of loudspeakers, also called microspeakers, are used in small electronic devices like mobile phones or handheld computers. To minimize the size of a microspeaker there are some restrictions to the materials and the soundquality. The materials have to be light but also robust. Therefore the basket and the membrane are generally made of plastic materials. In microspeakers the spider is omitted because of the lack of space inside the microspeaker. This leads to some difficulties in the axial alignment of the voice-coil. Nowadays the outer shape of a lot of microspeakers is rectangular, for easier integration into electronic devices. Microspeakers are often driven below their resonance frequency; this region is dominated by the suspension compliance and viscoelastic effects are relevant to consider [3]. This also generates new challenges for the designers. In figure 2 you can see a typical structure of an electrodynamic microspeaker.

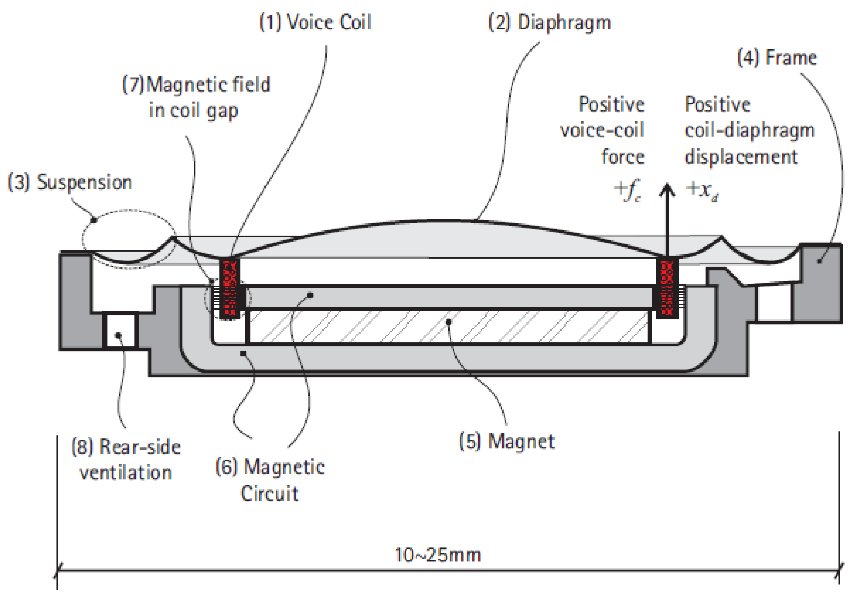


Figure 2: Basic components of an electrodynamic microspeaker. [5]

2.2 Thiele-Small-Parameters

An electrodynamic loudspeaker can be described by a set of parameters which represent the main characteristics of the behaviour of a loudspeaker. This parameters tend to represent real physical values like the mass of the membrane or the electrical inductance of the voice-coil. Thiele and Small have found a minimum set of parameters which describe a loudspeaker properly [12]. The Thiele-Small-Parameters are measured at small signal level. A list of these Thiele-Small-Parameters can be seen in Table 1.

L_e	Electrical inductance of the voice coil [H]
R_e	Electrical resistance [Ohm]
BL	Force-factor [Tm]
M_{ms}	Mass of the membrane + coil + acoustic load [kg]
C_{ms}	Compliance of the suspension [m/N]
R_{ms}	Mechanical resistance [Ns/m]
S_d	Projected area of the membrane [m²]
f_0	Resonance frequency [Hz]
V_{as}	Equivalent compliance volume [m³]
Q_{es}	Electrical quality factor
Q_{ms}	Mechanical quality factor
Q_{ts}	Total quality factor

Table 1: List of the Thiele-Small-Parameters.

The electrical inductance L_e is indicated at 1 kHz because it is frequency dependent. The mass M_{ms} is compound of the mass of the membrane, the mass of the voice coil and the acoustic mass which indicates the resistance of the moving membrane in air. The equivalent compliance volume V_{as} is the air volume which has the same compliance as the suspension with respect to the projected area S_d.

The Thiele-Small-Parameters describe the behaviour of the loudspeaker near the resonance frequency. The aim of knowing the loudspeaker parameters is to be able to simulate the behaviour of the loudspeaker and to simulate the properties of the membrane like position, velocity and acceleration. Also one can simulate the impedance of the input and the sound output of the loudspeaker. These parameters take also the enclosure of the loudspeaker into account.

There are several methods to measure the Thiele-Small-Parameters. One way is to measure the input impedance of the loudspeaker. There you can see the resonance frequency and the quality factor. With them you can calculate the position of the membrane and the missing parameters. Another way is to measure the velocity or position of the membrane and calculate the parameters out of them. This measurement can be done in a vacuum surrounding to neglect the influence of the acoustic mass.

2.3 Equivalent network of the traditional loudspeaker model

The traditional model uses the Thiele-Small-Parameters to describe a loudspeaker [13, 8]. The model consists of three parts. These parts represent the characteristics in the electrical, mechanical and acoustic domain. The parameters are held linear to be able to calculate the influences of the different domains separately.

The membrane is modelled with three mechanical components. A mass m, a spring (k) and a dashpot R_m which are in parallel connection. In figure 3 you can see the mechanical setting of the membrane.

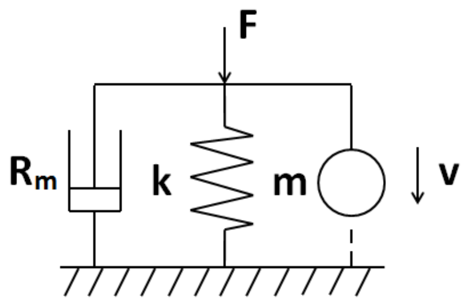


Figure 3: Mechanical model of the membrane.

The electrical components consist of a voltage source u, an electrical resistance R_e and an inductance L_e. We are interested in the low frequency behaviour of the loudspeaker, therefore we can neglect the influence of the inductance L_e because it only influences the high frequency range. The simplified model of the electrical voltage source can be seen in figure 4.

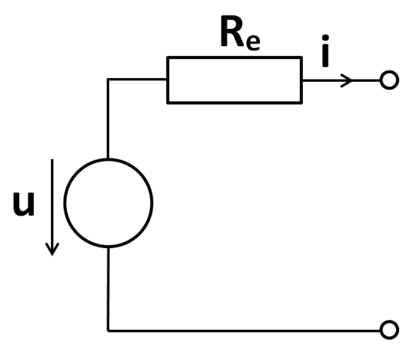


Figure 4: Simplified model of the electrical part.

The acoustic components consist of the acoustic load m_a, which is the mass of the moving air, and the acoustic radiation resistance R_a. They can be neglected if we assume that we simulate the loudspeaker in a vacuum environment. This method is often used when measuring the excursion. Someone can then calculate the acoustic components by comparing a measurement in vacuum to a measurement in air-filled surroundings.

We can transform the mechanical components into the electrical domain and vice versa by using the force-current (FI)-analogy. Therefore we use the transformation constant BL which is called the force-factor and describes what force gets imposed on the current carrying voice coil by the magnet. It depends on the magnetic field and the length of the current carrying conductor.

$$L = \frac{BL^2}{k}; \quad C = \frac{m}{BL^2}; \quad R = \frac{BL^2}{R_m}; \quad u_2 = BL \cdot v_2 = BL \cdot \frac{dx_2}{dt} \tag{1}$$

To get an equivalent network in the electrical domain for the mechanical and electrical components, seen in figure 5, we use the force-current analogy. The consideration of the acoustic components can be avoided by assuming vacuum environment.

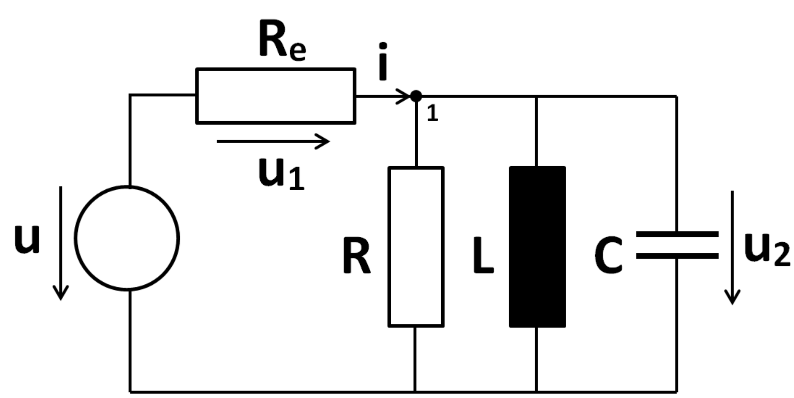


Figure 5: Equivalent network in the electrical domain.

2.4 Problems with the traditional loudspeaker model

There are a few restrictions to the traditional model which make this model only valid under distinct conditions. The traditional model is a simplified model which takes only the main characteristics of loudspeaker behaviour into account. One main restriction is the linearity of the parameters. To be able to separate the model into the electrical, mechanical and acoustic domain it has to be assumed that the parameters are in a linear range. This is only a good approximation for small signal levels. For large signal levels the loudspeaker can have a nonlinear behaviour. Some of the parameters, which are assumed linear, are actually nonlinear. Examples for this are the force-factor BL and the spring constant k [2, 10].

Another problem is that the traditional model is only of the order two. It is a simple linear model with as few parameters as possible but there are effects which are not represented with this model. The traditional model is focused on describing the region near the resonance frequency. With the model order two you can only model a single resonance and no other shapes of the frequency response. To be able to model for example the creep effect, which causes an increase of the excursion in the low frequency range (see figure 18, 19), you have to increase the order of your model (see chapter 4.3).

3 Viscoelasticity and Creep

Viscoelasticity is a material property, which combines the properties from Elasticity and Viscosity. The relevant physical quantities are stress and strain. The different effects of viscoelasticity on materials are Creep, Recovery and Relaxation. Creep is the effect, which is most relevant to consider in loudspeaker design.

3.1 Stress

Stress is a physical quantity that appears inside of material. It describes the interaction of the forces of adjacent particles to each other. The stress can be measured by the force divided by the area on which it acts. Stress is defined as the average force per unit area that particles of a material exert to their adjacent particles.

$$\sigma = \frac{F}{A} \tag{2}$$

The unit of stress is newton per square meter ($\frac{N}{m^2}$) or Pascal (Pa). Pressure is measured in the same units but it is defined differently. Pressure is the applied force on an area. Stress instead can be seen as the opposite force inside the material, which withstands the applied pressure. Therefore they have the same magnitude but an opposite sign. Pressure is used positive when it pushes and stress is used positive when it pulls.

There are different states of stress which differ in their direction of the force to the area. There is tension if the force is pulling apart a material. The opposite, where a material is pressed together, is called compression. If the force is parallel instead of normal to the area, we speak from shear stress. These states of stress can be defined in one dimension or in more. Then we speak of a stress tensor, which is a matrix representation of the stresses in the different dimensions.

3.2 Strain

Strain is the deformation of a material due to stress. It is a measure of the intensity of deformation at a point. It is representing the displacement between particles in the body relative to a reference length. Strain is a dimensionless quantity, since it is a ratio between lengths or volumes. Depending on the direction strain can be normal strain or shear strain.
Normal strain is defined as:

$$\varepsilon = \frac{\Delta L}{L} = \frac{l - L}{L} \tag{3}$$

Where L is the original length of a body and l is the length after deformation.

3.3 Elasticity

Materials, which return to their initial state after being deformed, are called elastic materials. These materials can be described by their elastic modulus and their elastic limit. Hooke's law connects the relation between stress and strain over the elastic modulus E (Young's modulus).

$$\sigma = E \cdot \varepsilon \tag{4}$$

This relation can also be stated for a spring like:

$$F = k \cdot x \tag{5}$$

Where F is the implied force, x is the displacement and k is the spring constant.

If the strain is small and the deformation does not exceed the elastic limit, these equations are linear. A low elastic modulus signifies that the material is easy to deform.

3.4 Viscosity

A property of fluids is viscosity. It is a measure of its resistance to gradual deformation due to stress. It also can be described through the concept of thickness for liquids. A fluid flows, which means particles of different layers of the fluid are gliding over each other with different velocities. The thicker or more viscous a fluid is, the slower it flows.

In the context of a fluid flowing between two parallel plates, where one is fixed and the other is moving with a constant speed v, a differential expression for viscous forces is:

$$\tau = \eta \frac{\partial v}{\partial y} \tag{6}$$

where η is the viscosity factor, $\tau = \frac{F}{A}$ and $\frac{\partial v}{\partial y}$ is the derivative of the velocity of the fluid v with respect to the path in direction perpendicular to the plates.

3.5 Viscoelasticity

Under viscoelasticity one understands the combination of elastic and viscous properties of a material. The strain reaction to an applied stress becomes time-dependent. For linear viscoelasticity the properties can be separated into the two components of elastic and viscous behaviour. In figure 6 the viscoelastic properties creep and recovery are shown.

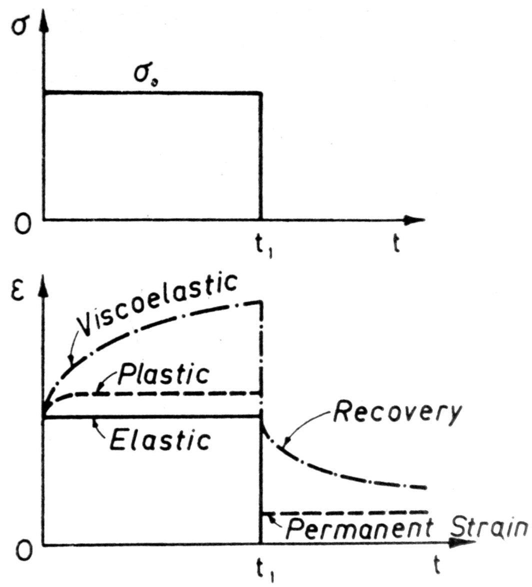


Figure 6: Viscoelastic properties creep and recovery. [7]

3.6 Creep

Creep is a phenomenon, which originates from the viscoelastic property of a material. It is a slow continuous deformation with time. The strain of a material increases with time while a constant stress is applied. The strain inflicted by a stress load gets time and temperature dependent [6, 4]. At room temperature materials like metals or ceramics don't creep, but some polymers do. Due to the temperature dependency materials creep more at high temperatures, which mean temperatures near the melting point or glasstemperature of the materials. Metals and ceramics start to creep at temperatures about 30% to 50% of the melting temperature. In figure 7, materials with their different melting points are shown.

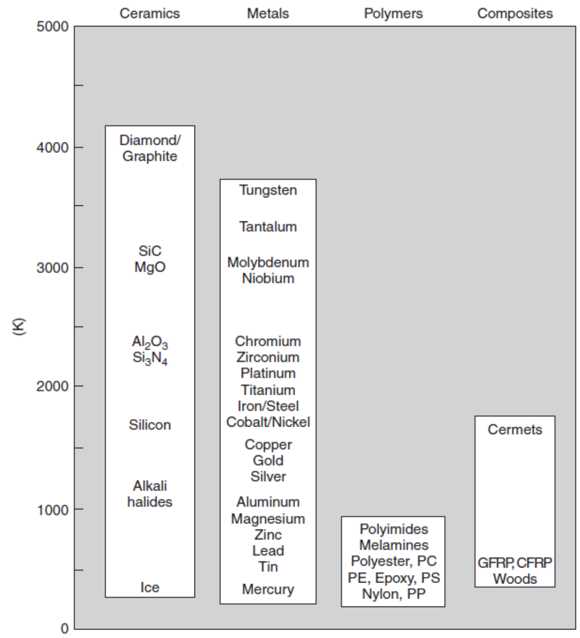


Figure 7: Melting or softening temperature of different materials. [4]

Figure 6 showed an example where constant stress is applied for a limited time. There you can see the primary and secondary stage of creep followed by a recovery phase where the stress got removed.

In figure 8 three stages of creep are shown, which are the results of a typical creep test where a material gets loaded with a constant stress for a long time while maintaining the same temperature. After applying a constant stress to a material it reacts with an initial elastic jump in the strain. The strain rises fast in the primary stage, it continues rising linearly in the secondary stage and increases rapidly again in the tertiary stage. The primary stage is characterized by the strain hardening of the material. In the second stage the strain hardening and the thermal softening get in balance; therefore the creep rate stays constant. In the tertiary stage the material gets weaker until it reaches a final limit or it breaks.

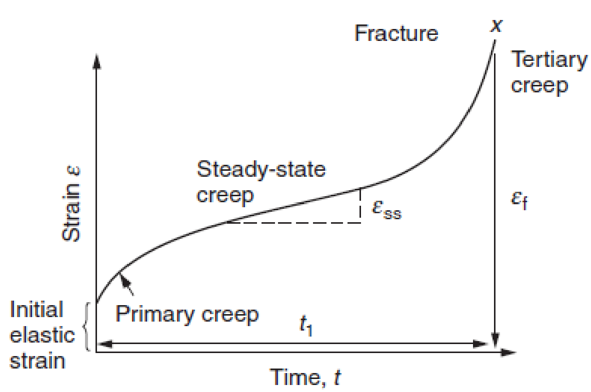


Figure 8: The three stages of creep. [4]

By appling an oscillatory stress to a linearly viscoelastic material the strain response will be an oscillation at the same frequency as the stress but lagging behind by a phase angle δ, see figure 9. The stiffness of a material gets dependent on the application rate of the inflicted stress load.

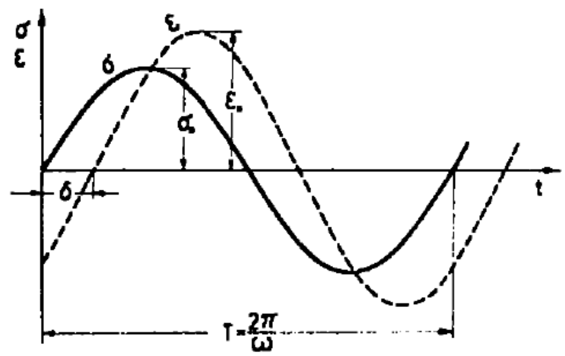


Figure 9: Oscillating stress σ, strain ε and phase lag δ. [7]

3.7 Creep in loudspeakers

Creep is depending on the material properties. Some polymers are able to creep at room temperature, while metals need a high temperature for creeping. In loudspeakers the suspension is normally made out of polymers or compound layers of polymers. Therefore creep can be considered relevant [1]. Due to the temperature dependency, the creep effect can become more problematic during long operations of a loudspeaker because of heat development of the electronic device. Loudspeakers are generally driven with periodic signals, where the membrane is deflected in both directions (see figure 9).

For low frequencies the displacement of the membrane rises. The force gets implied slowly so that the material has less instantaneous resistance to the force. For high frequencies the material has a higher inertia and the direction of the force changes so rapidly that the creep effect does not come into account.

A higher displacement can lead to problems because the maximal possible displacement is one of the limiting factors of a loudspeaker. The reasons are place restrictions due to the enclosure of the speaker, microspeakers are often used beneath a cover, and the nonlinear effects, which occur with high displacements. But high displacement is needed for a high sound pressure level of the speaker.

4 Loudspeaker models including creep

This section is about three different models to describe the viscoelastic creep effect. There are two models consisting of an elastic spring and a viscous damper, the Maxwell model, where the components are in series, and the Kelvin-Voigt model, where the components are in parallel. A combination of these two models is the often used Standard Linear Solid (SLS) model. The SLS model is able to model the viscoelastic creep effect in a simple way.

4.1 Maxwell model

The Maxwell model for a material consists of an elastic spring and a viscous damper in series. The two components are assumed to react ideally, like a Hookean spring and a Newtonian damper. In figure 10, the arrangement of the components is shown.

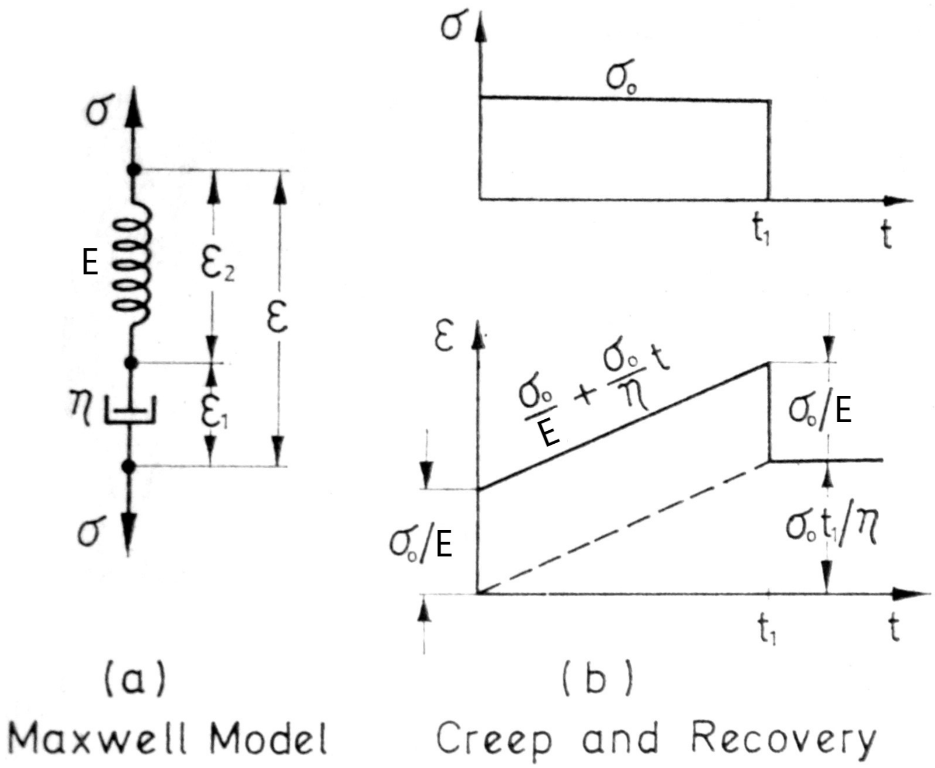


Figure 10: Maxwell model with stress and strain curves for creep and recovery. [7]

The stress-strain relation of the spring is:

$$\sigma = E \cdot \varepsilon \tag{7}$$

The stress-strain relation of the dashpot is:

$$\sigma = \eta \frac{d\varepsilon}{dt} \tag{8}$$

The total stress and strain can be calculated as followed:

$$\sigma_{total} = \sigma_{spring} = \sigma_{damper} \tag{9}$$

$$\varepsilon_{total} = \varepsilon_{spring} + \varepsilon_{damper} \tag{10}$$

To get the strain rate relation we combine equations (7), (8) and (10):

$$\frac{d\varepsilon_{total}}{dt} = \frac{d\varepsilon_{spring}}{dt} + \frac{d\varepsilon_{damper}}{dt} = \frac{1}{E}\frac{d\sigma}{dt} + \frac{\sigma}{\eta} \tag{11}$$

Where E is the elastic modulus and η is the viscosity coefficient.

Integrating equation (11) and applying a constant stress σ_0 at $t = 0$ leads to following equation for creep:

$$\boxed{\varepsilon(t) = \frac{\sigma_0}{E} + \frac{\sigma_0}{\eta}t} \tag{12}$$

A problem with this model is that it does not model creep accurately (compare figure 6 and 10).

4.2 Kelvin-Voigt model

This model also consists of an elastic spring and a viscous damper, but they are connected in parallel, like shown in figure 11.

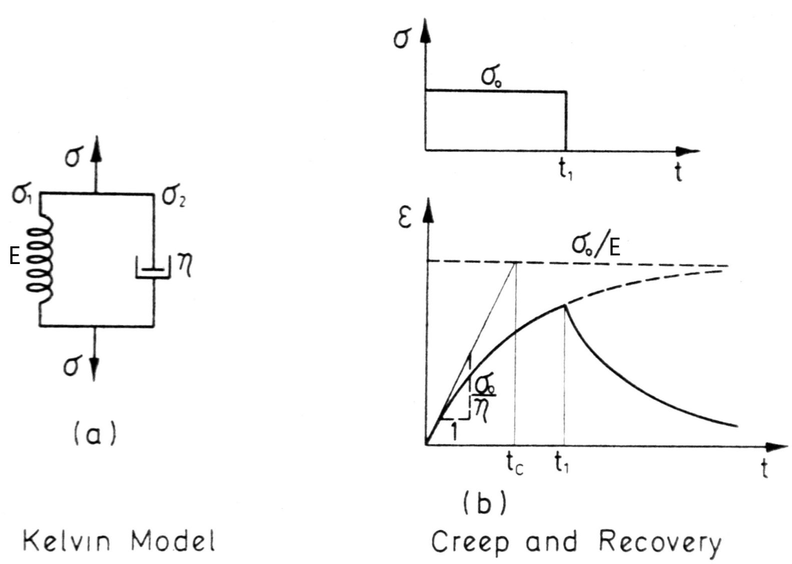


Figure 11: Kelvin-Voigt model with stress and strain curves for creep and recovery. [7]

The total stress and strain can be calculated as followed:

$$\sigma_{total} = \sigma_{spring} + \sigma_{damper} \tag{13}$$

$$\varepsilon_{total} = \varepsilon_{spring} = \varepsilon_{damper} \tag{14}$$

Combining equations (7), (8) and (13) leads to following equation:

$$\frac{d\varepsilon}{dt} + \frac{E}{\eta}\varepsilon = \frac{1}{\eta}\sigma \tag{15}$$

Solving this equation leads to the following form for creep under a constant stress σ_0 applied at $t = 0$:

$$\boxed{\varepsilon(t) = \frac{\sigma_0}{E}\left(1 - e^{-\frac{E}{\eta}t}\right)} \tag{16}$$

The Kelvin-Voigt model also does not describe creep and recovery accurately (compare figure 6 and 11).
The retardation time $t_c = \frac{\eta}{E}$ is the time at which the strain would cross the asymptotic value $\frac{\sigma_0}{E}$ if the strain were to increase at its initial rate $\frac{\sigma_0}{\eta}$.

The stress with duration t_1 can be represented by two step inputs:

$$\sigma(t) = \sigma_0 H(t) - \sigma_0 H(t - t_1) \tag{17}$$

H(t) is the Heavyside or unit step function and is defined as:

$$H(t) = \left\{ \begin{array}{c} 1,\ t \geq 0 \\ 0,\ t < 0 \end{array} \right\} \tag{18}$$

The resulting response is then:

$$\varepsilon(t) = \frac{\sigma_0}{E}\left(1 - e^{-\frac{E}{\eta}t}\right) H(t) - \frac{\sigma_0}{E}\left(1 - e^{-\frac{E}{\eta}(t-t_1)}\right) H(t - t_1) \tag{19}$$

For the creep phase from $0 < t < t_1$ equation (16) is valid.
For the recovery phase from $t > t_1$ following equation results:

$$\boxed{\varepsilon(t) = \frac{\sigma_0}{E} e^{-\frac{E}{\eta}t}\left(e^{\frac{E}{\eta}t_1} - 1\right)} \tag{20}$$

4.3 Standard Linear Solid model (SLS)

A combination of the Maxwell and the Kelvin-Voigt model is the Standard Linear Solid model. The model consists of two springs and a damper like shown in figure 12. The advantage of the SLS model is that it can be used to simulate creep and recovery accuratley (compare figure 6 and 13). It combines properties from the Maxwell and the Kelvin-Voigt model (see figure 13).

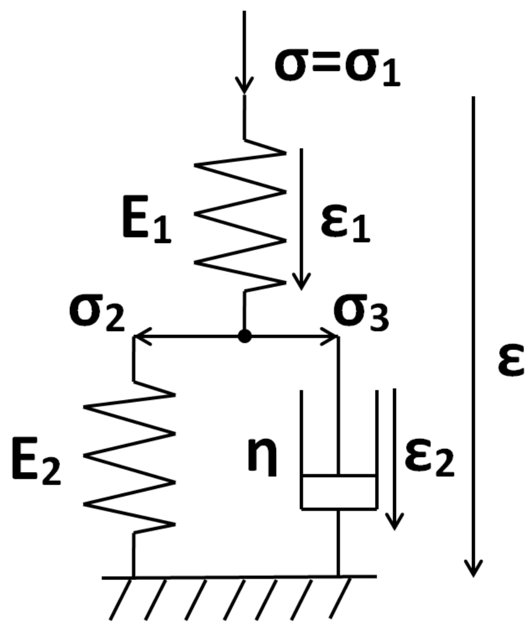


Figure 12: Standard Linear Solid model.

Stress and strain can be calculated similar to the previous models using equations (7), (8).
To calculate the differential equation from figure 12 we start with:

$$\sigma = \sigma_2 + \sigma_3$$

$$\varepsilon = \varepsilon_1 + \varepsilon_2$$

We express ε_2 as:

$$\varepsilon_2 = \varepsilon - \varepsilon_1 = \varepsilon - \frac{1}{E_1}\sigma$$

Then we use this equation to express σ_2 and σ_3.

$$\sigma_2 = E_2 \varepsilon_2 = E_2 \varepsilon - \frac{E_2}{E_1} \sigma$$

$$\sigma_3 = \eta \frac{d\varepsilon_2}{dt} = \eta \frac{d\varepsilon}{dt} - \frac{\eta}{E_1} \frac{d\sigma}{dt}$$

We combine these equations to get following stress-strain relation:

$$\sigma = E_2 \varepsilon - \frac{E_2}{E_1} \sigma + \eta \frac{d\varepsilon}{dt} - \frac{\eta}{E_1} \frac{d\sigma}{dt}$$

$$\frac{d\varepsilon}{dt} + \frac{E_2}{\eta} \varepsilon = \frac{E_1 + E_2}{E_1 \eta} \sigma + \frac{1}{E_1} \frac{d\sigma}{dt} \tag{21}$$

Solving this equation and applying a constant stress σ_0 at $t = 0$ leads to following equation for creep:

$$\boxed{\varepsilon(t) = \frac{\sigma_0}{E_1} + \frac{\sigma_0}{E_2} \left(1 - e^{-\frac{E_2}{\eta} t}\right)} \tag{22}$$

This equation is the superposition of the solutions for a spring (7) and the Kelvin-Voigt model (16). To get the recovery phase we use equation (17) and (19) which results in following equation for $t > t_1$:

$$\boxed{\varepsilon(t) = \frac{\sigma_0}{E_2} e^{-\frac{E_2}{\eta} t} \left(e^{\frac{E_2}{\eta} t_1} - 1\right)} \tag{23}$$

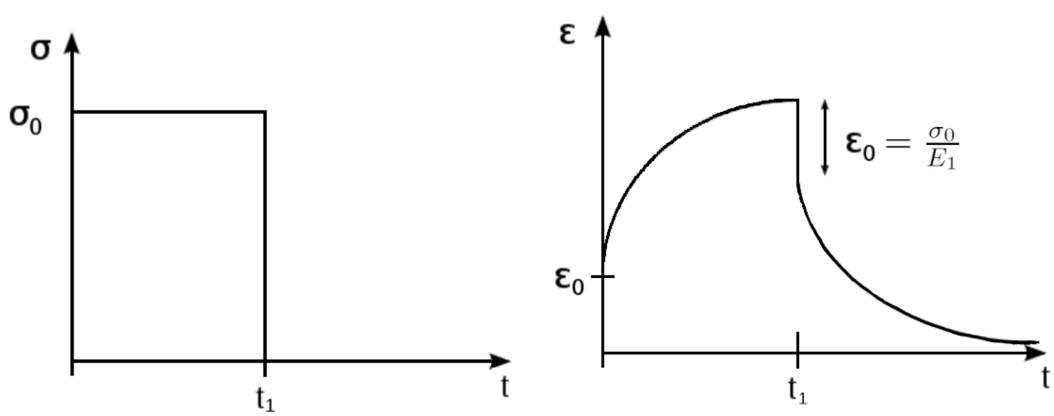


Figure 13: Standard Linear Solid model with stress and strain curves for creep and recovery.

5 Implementation of the SLS model

In this section we are going to implement a practical solution for the Standard Linear Solid model to simulate the excursion of a loudspeaker membrane. Other loudspeaker models which consider suspension creep are usually described in frequency domain [11, 9, 3]. We want to have a model in time domain; therefore we implement the SLS model in form of a differential equation. The advantage of the representation in time domain is that we can directly calculate the time response to a given signal (see figure 17) and therefore are able to consider nonlinearities, like the nonlinearity of the force-factor BL. By using the SLS model we take the influence of the creep effect into account. Then we describe the calculation procedure in more detail.

We begin with the derivation of the differential equation out of the equivalent circuit for the traditional loudspeaker model. Then we derive the differential equation for the SLS model in a direct and an indirect way. We take the nonlinearity of the force-factor BL into account and solve the differential equations using the Matlab ode-solver.

5.1 Derivation of the differential equation of the traditional loudspeaker model

Our goal is to simulate the excursion of the loudspeaker membrane. We are putting together a loudspeaker model by using and simplifying the parts of the different domains, described in chapter 2.3.

For the equivalent network in figure 5 we can use Kirchhoff's current law and Kirchhoff's voltage law to set the equations for the network. With those we can calculate a differential equation for the voltage u_2, which later can be transformed into the velocity v_2 and displacement x_2.

$$u = u_1 + u_2$$

$$i = i_R + i_L + i_C$$

$$i_R = \frac{1}{R} u_2; \quad i_L = \frac{1}{L} \int u_2 \, dt; \quad i_C = C \frac{du_2}{dt}$$

$$i = \frac{1}{R} u_2 + \frac{1}{L} \int u_2 \, dt + C u_2'$$

$$u_1 = R_e \cdot i = u - u_2$$

$$i = \frac{1}{R_e} u - \frac{1}{R_e} u_2$$

When we combine the two ways of calculating i we get following equation:

$$\frac{1}{R}u_2 + \frac{1}{L}\int u_2\,dt + Cu_2' = \frac{1}{R_e}u - \frac{1}{R_e}u_2$$

$$Cu_2' + \left(\frac{1}{R} + \frac{1}{R_e}\right)u_2 + \frac{1}{L}\int u_2\,dt = \frac{1}{R_e}u$$

$$u_2' + \left(\frac{1}{RC} + \frac{1}{R_eC}\right)u_2 + \frac{1}{LC}\int u_2\,dt = \frac{1}{R_eC}u$$

Now we transform the equation partially into the mechanical domain. The mechanical parts get transformed and the electrical parts stay in the electrical domain. For this transformation we use the equations (1).

$$BL \cdot x_2'' + \left(\frac{R_m}{BL^2}\frac{BL^2}{m} + \frac{1}{R_e}\frac{BL^2}{m}\right)BL \cdot x_2' + \frac{k}{BL^2}\frac{BL^2}{m}BL \cdot x_2 = \frac{1}{R_e}\frac{BL^2}{m}u$$

$$\boxed{x_2'' + \left(\frac{R_m}{m} + \frac{BL^2}{R_em}\right)x_2' + \frac{k}{m}x_2 = \frac{BL}{R_em}u} \tag{24}$$

We calculated a differential equation which describes the displacement x_2 with mechanical and electrical parameters and a voltage input signal.

5.2 Derivation of the differential equation of the SLS model

To be able to take the creep effect into account we extend the traditional loudspeaker model. We replace the spring (k) with a spring (k_1) in series with another spring (k_2) parallel to a dashpot R_{m1}. This structure is also known as the Standard Linear Solid (SLS) model. In figure 14 we see the modified mechanical parts.

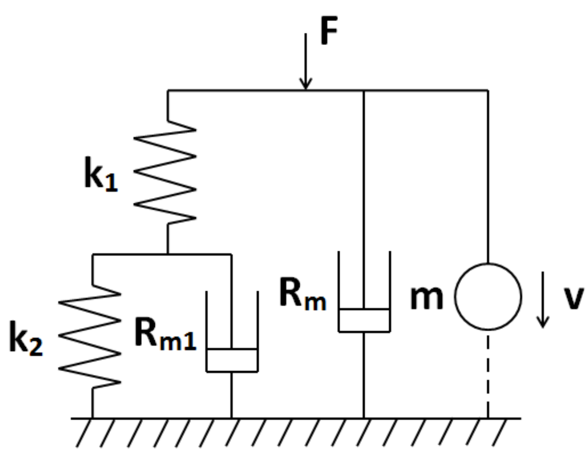


Figure 14: Modified mechanical parts.

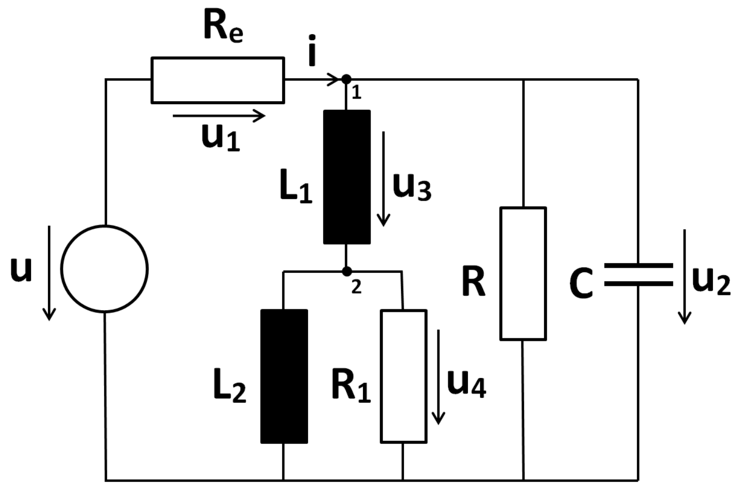


Figure 15: Combined equivalent network in electrical domain for the SLS model.

In figure 15 we see the equivalent network in the electrical domain for the SLS model. Like with the traditional loudspeaker model we use Kirchhoff's laws to set the equations and then calculate a differential equation for the displacement x_2. We start by using the nodal rule at node 2 to get an expression for u_3.

$$i_{L_1} = i_{L_2} + i_{R_1}$$

$$\frac{1}{L_1}\int u_3 dt = \frac{1}{L_2}\int u_4 dt + \frac{1}{R_1}u_4$$

$$u_3 = \frac{L_1}{L_2}u_4 + \frac{L_1}{R_1}u_4' \tag{25}$$

We use the mesh rule to get an expression for i and i'.

$$u_2 = u - u_1 = u - R_e \cdot i$$

$$i = \frac{1}{R_e}u - \frac{1}{R_e}u_2$$

$$i' = \frac{1}{R_e}u' - \frac{1}{R_e}u_2'$$

Then we use the nodal rule at node 1 to get an expression for i_{L_1}'.

$$i = i_{L_1} + i_R + i_C$$

$$i_{L_1}' = i' - i_R' - i_C'$$

We use the expression for i'_{L_1} and the mesh rule to get an expression for u_4.

$$u_4 = u_2 - u_3 = u_2 - L_1 i'_{L_1} = u_2 - L_1\left(i' - i'_R - i'_C\right) = u_2 - L_1\left(\frac{1}{R_e}u' - \frac{1}{R_e}u'_2 - \frac{1}{R}u'_2 - Cu''_2\right)$$

We set up an equation for u_2 which is dependent on u_4.

$$u_2 = u_3 + u_4 = \frac{L_1}{L_2}u_4 + \frac{L_1}{R_1}u'_4 + u_4 = \left(1 + \frac{L_1}{L_2}\right)u_4 + \frac{L_1}{R_1}u'_4 \tag{26}$$

Then we combine the equations to get a differential equation for u_2.

$$u_2 = \left(1 + \frac{L_1}{L_2}\right)\left(u_2 - \frac{L_1}{R_e}u' + \frac{L_1}{R_e}u'_2 + \frac{L_1}{R}u'_2 + L_1Cu''_2\right) + \frac{L_1}{R_1}\left(u'_2 - \frac{L_1}{R_e}u'' + \frac{L_1}{R_e}u''_2 + \frac{L_1}{R}u''_2 + L_1Cu'''_2\right)$$

$$\begin{aligned}\frac{L_1^2C}{R_1}u'''_2 + \left(L_1C + \frac{L_1^2C}{L_2} + \frac{L_1^2}{R_1R_e} + \frac{L_1^2}{R_1R}\right)u''_2 + \left(\frac{L_1}{R_e} + \frac{L_1^2}{L_2R_e} + \frac{L_1}{R} + \frac{L_1^2}{L_2R} + \frac{L_1}{R_1}\right)u'_2 + \frac{L_1}{L_2}u_2 = \dots \\ = \left(\frac{L_1}{R_e} + \frac{L_1^2}{L_2R_e}\right)u' + \frac{L_1^2}{R_1R_e}u''\end{aligned}$$

$$\begin{aligned}u''_2 + \left(\frac{R_1}{L_1} + \frac{R_1}{L_2} + \frac{1}{CR_e} + \frac{1}{CR}\right)u'_2 + \left(\frac{R_1}{L_1CR_e} + \frac{R_1}{L_2CR_e} + \frac{R_1}{L_1CR} + \frac{R_1}{L_2CR} + \frac{1}{L_1C}\right)u_2 + \dots \\ + \frac{R_1}{L_1L_2C}\int u_2 dt = \left(\frac{R_1}{L_1CR_e} + \frac{R_1}{L_2CR_e}\right)u + \frac{1}{CR_e}u'\end{aligned}$$

Now we transform the equation for u_2 partially into the mechanical domain like in section 5.1.

$$\boxed{\begin{aligned}x'''_2 + \left(\frac{k_1}{R_{m1}} + \frac{k_2}{R_{m1}} + \frac{BL^2}{mR_e} + \frac{R_m}{m}\right)x''_2 + \dots \\ + \left(\frac{k_1BL^2}{R_{m1}mR_e} + \frac{k_2BL^2}{R_{m1}mR_e} + \frac{k_1R_m}{R_{m1}m} + \frac{k_2R_m}{R_{m1}m} + \frac{k_1}{m}\right)x'_2 + \frac{k_1k_2}{R_{m1}m}x_2 = \dots \\ = \left(\frac{k_1BL}{R_{m1}mR_e} + \frac{k_2BL}{R_{m1}mR_e}\right)u + \frac{BL}{mR_e}u'\end{aligned}} \tag{27}$$

We have a differential equation for the displacement x_2.

5.3 Alternative derivation of the differential equation of the SLS model

There is an alternative way to derive the displacement x_2, where we first find a differential equation for x_4 and then calculate x_2 afterwards. This way we can solve a different differential equation, which improves the calculation time by a third to a half. We use the same equivalent network from figure 15. As in section 5.2 we start by getting expressions for u_3 [equation(25)] and u_2 [equation(26)].
This time we are looking for two expressions for i.

$$i = i_{L_1} + i_R + i_C = \frac{1}{L_1}\int u_3 dt + \frac{1}{R}u_2 + Cu_2'$$

$$i = \frac{1}{L_1}\int\left(\frac{L_1}{L_2}u_4 + \frac{L_1}{R_1}u_4'\right)dt + \frac{1}{R}\left[\left(1+\frac{L_1}{L_2}\right)u_4 + \frac{L_1}{R_1}u_4'\right] + C\left[\left(1+\frac{L_1}{L_2}\right)u_4' + \frac{L_1}{R_1}u_4''\right]$$

$$i = \frac{CL_1}{R_1}u_4'' + \left(\frac{L_1}{RR_1} + C + \frac{CL_1}{L_2}\right)u_4' + \left(\frac{1}{R_1} + \frac{1}{R} + \frac{L_1}{L_2R}\right)u_4 + \frac{1}{L_2}\int u_4 dt$$

$$i = \frac{1}{R_e}u - \frac{1}{R_e}u_2 = \frac{1}{R_e}u - \frac{1}{R_e}\left[\left(1+\frac{L_1}{L_2}\right)u_4 + \frac{L_1}{R_1}u_4'\right] = \frac{1}{R_e}u - \left(\frac{1}{R_e} + \frac{L_1}{L_2R_e}\right)u_4 - \frac{L_1}{R_1R_e}u_4'$$

We combine the two expressions to get a differential equation for u_4.

$$\frac{CL_1}{R_1}u_4'' + \left(\frac{L_1}{RR_1} + C + \frac{CL_1}{L_2} + \frac{L_1}{R_1R_e}\right)u_4' + \left(\frac{1}{R_1} + \frac{1}{R} + \frac{L_1}{L_2R} + \frac{1}{R_e} + \frac{L_1}{L_2R_e}\right)u_4 + \frac{1}{L_2}\int u_4 dt = \frac{1}{R_e}u$$

$$u_4'' + \left(\frac{1}{CR} + \frac{R_1}{L_1} + \frac{R_1}{L_2} + \frac{1}{CR_e}\right)u_4' + \left(\frac{1}{CL_1} + \frac{R_1}{RCL_1} + \frac{R_1}{RCL_2} + \frac{R_1}{CL_1R_e} + \frac{R_1}{CL_2R_e}\right)u_4 + \dots$$
$$+\frac{R_1}{CL_1L_2}\int u_4 dt = \frac{R_1}{CL_1R_e}u$$

Now we transform the equation for u_4 partially into the mechanical domain like in section 5.1.

$$\boxed{\begin{aligned} &x_4''' + \left(\frac{R_m}{m} + \frac{k_1}{R_{m1}} + \frac{k_2}{R_{m1}} + \frac{BL^2}{mR_e}\right)x_4'' + \dots \\ &+\left(\frac{k_1}{m} + \frac{R_mk_1}{R_{m1}m} + \frac{R_mk_2}{R_{m1}m} + \frac{BL^2k_1}{R_{m1}mR_e} + \frac{BL^2k_2}{R_{m1}mR_e}\right)x_4' + \frac{k_1k_2}{R_{m1}m}x_4 = \frac{BLk_1}{R_{m1}mR_e}u \end{aligned}} \quad (28)$$

We have a differential equation for the displacement x_4. To calculate the displacement x_2 we use equation (26).

$$\boxed{x_2 = \left(1+\frac{k_2}{k_1}\right)x_4 + \frac{R_{m1}}{k_1}x_4'} \quad (29)$$

5.4 Force-factor BL and its nonlinearity

The force-factor BL is the transformation constant to transform an electrical variable into a mechanical or vice versa. It derives from the Lorentz force and its application in the example of the behaviour of an electrical wire inside a magnetic field, where the force F on the wire can be described by the current I through the wire times the magnetic field B times the length L of the wire. In the linear case BL is assumed to be a constant. However BL is dependent on the displacement of the loudspeaker membrane and therefore the differential equation becomes nonlinear.

We take a look at a measured BL curve in figure 16. We are interested in the region near the resonance frequency because that is the region with the highest displacement. To approximate the BL curve we use a polynomial fitting. If we take the region from -0.4 mm to 0.4 mm into account, which is a reasonable working point for a microspeaker, a second order polynomial is sufficient.

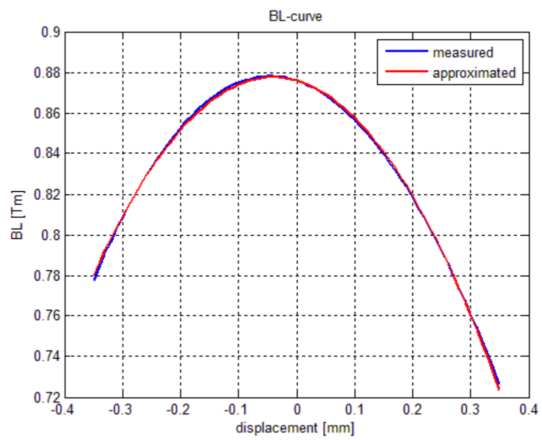


Figure 16: Comparison between measured and approximated BL curve.

If we describe BL as a polynomial of second order, which is dependent on the displacement x_2 we get an equation like:

$$BL\left(x_2\right) = c_0 + c_1 x_2 + c_2 x_2^2 \tag{30}$$

This representation of BL can be inserted into the differential equation (24) of the traditional loudspeaker model and equation (27) of the SLS model.
To get an equation where BL depends on x_4 we use formula (30) and (29).

$$BL(x_4) = c_0 + c_1 \left[\left(1 + \frac{k_2}{k_1}\right) x_4 + \frac{R_{m1}}{k_1} x_4'\right] + c_2 \left[\left(1 + \frac{k_2}{k_1}\right) x_4 + \frac{R_{m1}}{k_1} x_4'\right]^2$$

This representation of BL can be inserted into the differential equation (28) of the alternative SLS model.

When we calculate the parameters from the approximated curve seen in figure 16, we get:
$c_2 = -1.01$; $c_1 = -0.08$; $c_0 = 0.876$;

5.5 Solving the differential equations with MATLAB ode-solver

In literature there are a few approaches for creep models in frequency domain. Our approach is to solve the differential equation for the system in time domain. Therefore we use the MATLAB ode-solver. "ode" stands for ordinary differential equation.

The Matlab ode-solver solves problems of the form:

$$\frac{dx_1}{dt} = f_1\left(x_1, x_2, x_3, ..., t\right)$$
$$\frac{dx_2}{dt} = f_2\left(x_1, x_2, x_3, ..., t\right)$$
$$\frac{dx_3}{dt} = f_3\left(x_1, x_2, x_3, ..., t\right)$$
$$...$$

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \end{pmatrix} \qquad \vec{f} = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ \vdots \end{pmatrix}$$

$$\boxed{\frac{d\vec{x}}{dt} = \vec{f}\left(t, \vec{x}\right)}$$

We have to write the differential equation in the form of a system of differential equations.

For the traditional loudspeaker model the differential equation (24) is:

$$x'' + \left(\frac{R_m}{m} + \frac{BL^2}{R_e m}\right) x' + \frac{k}{m} x = \frac{BL}{R_e m} u(t)$$

This transformed into a system of differential equations is:

$$x_1 = x; \qquad x_2 = x'$$

$$x_3 = x'' = -\left(\frac{R_m}{m} + \frac{BL^2}{R_e m}\right) x_2 - \frac{k}{m} x_1 + \frac{BL}{R_e m} u(t) = -b \cdot x_2 - a \cdot x_1 + d \cdot u(t)$$

$$\boxed{\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -b \cdot x_2 - a \cdot x_1 + d \cdot u(t) \end{bmatrix}}$$

For the SLS model the differential equation (27) is:

$$x''' + \left(\frac{k_1}{R_{m1}} + \frac{k_2}{R_{m1}} + \frac{BL^2}{mR_e} + \frac{R_m}{m}\right)x'' + \left(\frac{k_1BL^2}{R_{m1}mR_e} + \frac{k_2BL^2}{R_{m1}mR_e} + \frac{k_1R_m}{R_{m1}m} + \frac{k_2R_m}{R_{m1}m} + \frac{k_1}{m}\right)x' + \dots$$
$$+\frac{k_1k_2}{R_{m1}m}x = \left(\frac{k_1BL}{R_{m1}mR_e} + \frac{k_2BL}{R_{m1}mR_e}\right)u(t) + \frac{BL}{mR_e}u'(t)$$

This transformed into a system of differential equations is:

$$x_1 = x; \qquad x_2 = x'; \qquad x_3 = x''; \qquad x_4 = x''' = \dots$$
$$= -\left(\frac{k_1}{R_{m1}} + \frac{k_2}{R_{m1}} + \frac{BL^2}{mR_e} + \frac{R_m}{m}\right)x_3 - \left(\frac{k_1BL^2}{R_{m1}mR_e} + \frac{k_2BL^2}{R_{m1}mR_e} + \frac{k_1R_m}{R_{m1}m} + \frac{k_2R_m}{R_{m1}m} + \frac{k_1}{m}\right)x_2 - \dots$$
$$-\frac{k_1k_2}{R_{m1}m}x_1 + \left(\frac{k_1BL}{R_{m1}mR_e} + \frac{k_2BL}{R_{m1}mR_e}\right)u(t) + \frac{BL}{mR_e}u'(t) = -c\cdot x_3 - b\cdot x_2 - a\cdot x_1 + d\cdot u(t) + e\cdot u'(t)$$

$$\boxed{\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_3 \\ -c\cdot x_3 - b\cdot x_2 - a\cdot x_1 + d\cdot u(t) + e\cdot u'(t) \end{bmatrix}}$$

For the alternative route of the SLS model over x_4 the differential equation of x_4 (28) is:

$$x''' + \left(\frac{R_m}{m} + \frac{k_1}{R_{m1}} + \frac{k_2}{R_{m1}} + \frac{BL^2}{mR_e}\right)x'' + \left(\frac{k_1}{m} + \frac{R_mk_1}{R_{m1}m} + \frac{R_mk_2}{R_{m1}m} + \frac{BL^2k_1}{R_{m1}mR_e} + \frac{BL^2k_2}{R_{m1}mR_e}\right)x' + \dots$$
$$+\frac{k_1k_2}{R_{m1}m}x = \frac{BLk_1}{R_{m1}mR_e}u(t)$$

This transformed into a system of differential equations is:

$$x_1 = x; \qquad x_2 = x'; \qquad x_3 = x''; \qquad x_4 = x''' = \dots$$
$$= -\left(\frac{R_m}{m} + \frac{k_1}{R_{m1}} + \frac{k_2}{R_{m1}} + \frac{BL^2}{mR_e}\right)x_3 - \left(\frac{k_1}{m} + \frac{R_mk_1}{R_{m1}m} + \frac{R_mk_2}{R_{m1}m} + \frac{BL^2k_1}{R_{m1}mR_e} + \frac{BL^2k_2}{R_{m1}mR_e}\right)x_2 - \dots$$
$$-\frac{k_1k_2}{R_{m1}m}x_1 + \frac{BLk_1}{R_{m1}mR_e}u(t) = -c\cdot x_3 - b\cdot x_2 - a\cdot x_1 + d\cdot u(t)$$

$$\boxed{\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_3 \\ -c\cdot x_3 - b\cdot x_2 - a\cdot x_1 + d\cdot u(t) \end{bmatrix}}$$

We use the ode15s-solver, which is a variable-order solver based on the numerical differentiation formulas of order 1 to 5. It is well suited for equations where the solution changes slowly. For our equations it works appropriately.

5.6 Calculation Procedure

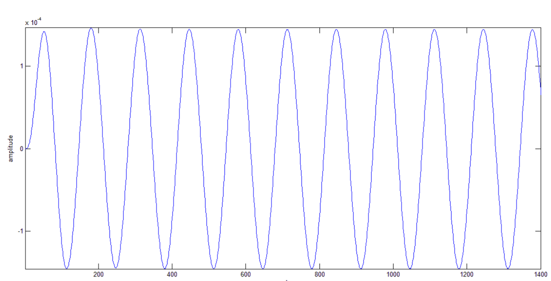
Here we describe the calculation procedure to simulate the excursion curve of the loudspeaker membrane.

We use a sine function for the excitation signal of the simulation. For every frequency we calculate the output of the simulation from an input sine function of the corresponding frequency. The frequencies are in the range from 10 Hz to 1000 Hz using a resolution of 100 frequency points in a logarithmic spacing. We use an amplitude of 1 Volt peak value and a 1 second long signal.

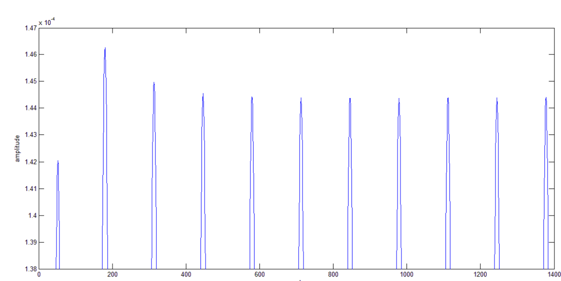
The parameters for the traditional loudspeaker model we set to:
$R_e = 7\ \Omega$; $m = 8.6e-05$ kg; $k = 1187$ N/m; $R_m = 0.09$ Ns/m; $BL = 0.876$ Tm
For the SLS model the paramater k gets substituted with the parameters:
$R_{m1} = 4$ Ns/m; $k_1 = 1110$ N/m; $k_2 = 2700$ N/m;
The parameter R_m gets changed to $R_m = 0.074$ Ns/m.

For the traditional loudspeaker model those are the Thiele-Small parameters. The parameters of the SLS model have no direct physical representation. We choose these parameters freely. For the nonlinear force-factor BL we use the coefficients, which were calculated in chapter 5.4.

Now we use the models derivated in chapter 5 to simulate the excursion. We use a sine function with one frequency, the parameters of the loudspeaker models and the coefficients of BL and put those into the differential equations of the models. The differential equations get solved in time domain by the MATLAB ode-solver. We get a time signal for the excursion, see figure 17. At the beginning of the time signal, seen in figure 17b, we see a transient before the signal reaches steady state. Then we calculate the maximum excitation in the steady state for this frequency. For the alternative SLS model we get a time signal for the excursion x_4 where we then calculate the overall excursion x_2 from.



(a) 331Hz



(b) zoomed transient at 331Hz

Figure 17: Time signal of the excursion.

6 Results

In this section we describe the results of the simulation. We describe the simulation with a nonlinear BL and compare it to a simulation with a constant BL.

6.1 Simulation with nonlinear BL

Figure 18 shows the results of the simulation of the excursion. The blue curve represents the traditional loudspeaker model which has an enhancement at the resonance frequency and a flat low frequency range. The black curve represents the SLS model, calculated the alternative way, which has also an enhancement at the resonance frequency but also an increase at low frequency range. The red curve represents a measured excursion curve which serves as our reference. The traditional model is optimized to represent the region around the resonance frequency but lacks in representing the low frequency range accurately. As can be seen, the SLS model is an improvement to be able to represent the low frequency range.

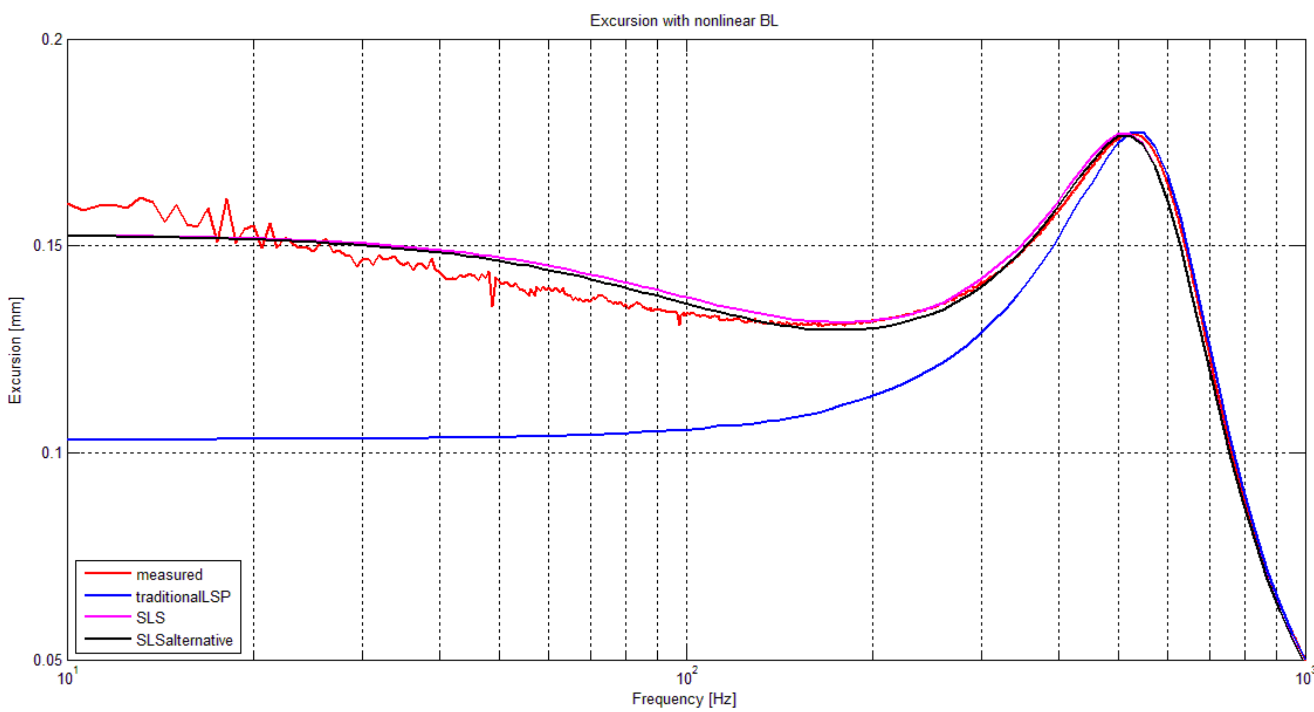


Figure 18: Excursion comparison with nonlinear BL.

Figure 18 includes both versions of the SLS model. The paths of calculation differ slightly but the results are quite the same.

6.2 Simulation with constant BL

To see the influence of the nonlinearity of the force-factor BL we did as well a simulation with constant BL. In figure 19 you can see the excitation curves of the constant BL case. The model parameters are the same as in the nonlinear BL case but the resulting amplitudes are a bit higher.

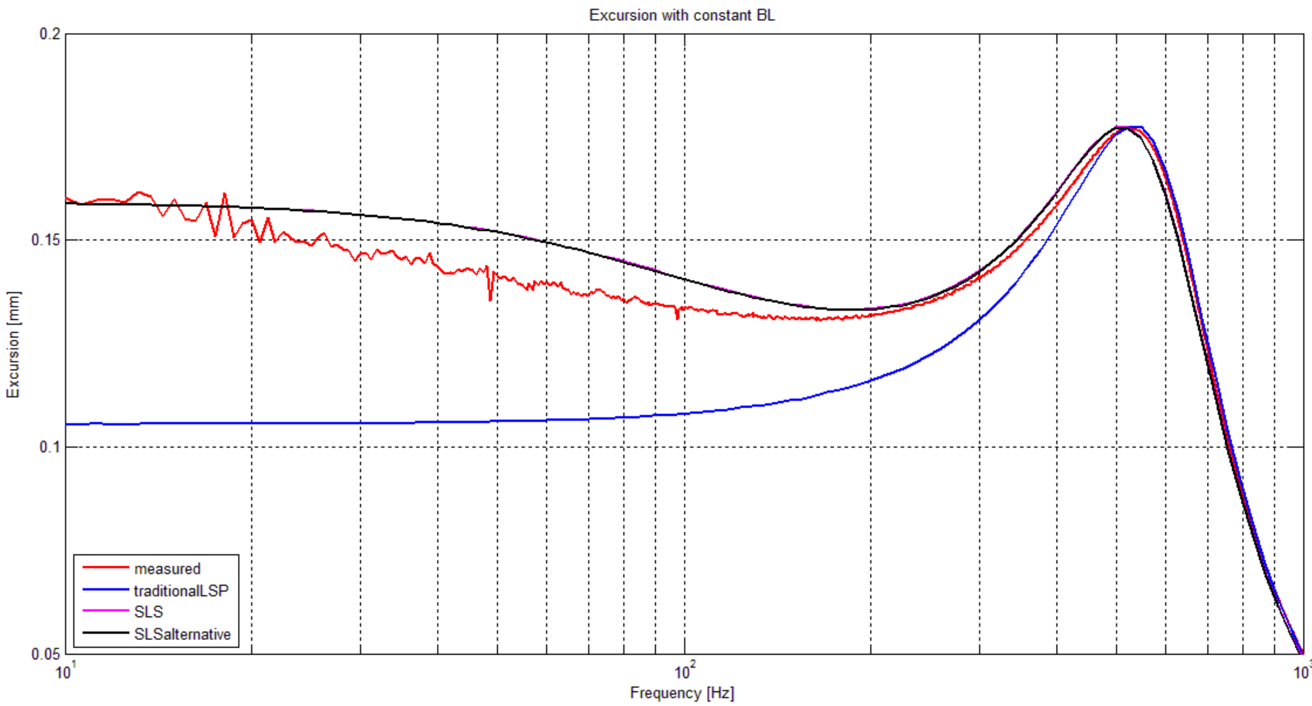


Figure 19: Excursion comparison with constant BL.

6.3 Program code description

Filename	Description
Simulation.m	loudspeaker simulation for 1 model
SimulationComparison.m	loudspeaker simulation comparing 3 models
TSPdatavacuum.mat	contains measurement data and Thiele-Small-parameters
BLcalculation.m	BL coefficient calculation and BL figure creation
CurveFitting.m	simple curve fitting and finding of parameters

Table 2: List of program files.

7 Conclusions

Suspension creep is quite complex to model due to its dependency on time, excursion and temperature. This project is just a first step of including suspension creep into the traditional loudspeaker model. By calculating the simulation in time domain it is possible to include nonlinearities like the force-factor BL. In chapter 6 it was shown that the SLS model is able to improve the simulation in the low frequency range. The alternative SLS model improves the calculation time by a third to a half by numerically solving a different differential equation.

Next steps in building upon the presented model are advanced parameter estimation and curve fitting. To analyse the nonlinear behaviour further, an examination of the total harmonic distortion (THD) is advised. It is expected that with higher input signals the excursion and therefore the nonlinearities would increase. In this project a microspeaker was used for the measurements, therefore it could be interesting to look into the difference in suspension creep compared to a standard sized speaker.

This project introduced an alternative way of simulating suspension creep in time domain to be able to consider nonlinearities.

References

[1] Finn Agerkvist, Knud Thorborg, and Carsten Tinggard. A Study of the Creep Effect in Loudspeaker Suspension. Audio Engineering Society, October 2008.

[2] Finn T. Agerkvist. Non-Linear Viscoelastic Models. Audio Engineering Society, October 2011.

[3] Finn T. Agerkvist and Tobias Ritter. Modeling Viscoelasticity of Loudspeaker Suspensions Using Retardation Spectra. Audio Engineering Society, November 2010.

[4] Michael F. Ashby and David R.H. Jones. *Engineering Materials 1: An Introduction to Properties, Applications and Design*. Butterworth Heinemann, 2011.

[5] Andrew Bright and Danmarks Tekniske Universitet Orsted-DTU. *Active Control of Loudspeakers: An Investigation of Practical Applications : PhD Thesis*. Orsted DTU, Acoustic Technology, Technical University of Denmark, 2002.

[6] Hal F. Brinson and L. Catherine Brinson. *Polymer Engineering Science and Viscoelasticity*. Springer, 2008.

[7] William N. Findley, James S. Lai, and Kasif Onaran. *Creep and Relaxation of Nonlinear Viscoelastic Materials*. North-Holland Publishing Company, 1976.

[8] Andreas Gaich. *Diskrete Lautsprechermodelle zur Simulation des Membranhubs und des Schalldrucks*. 2011.

[9] Holger Hiebel. Suspension Creep Models for Miniature Loudspeakers. Audio Engineering Society, April 2012.

[10] Wolfgang Klippel. Modeling the Large Signal Behavior of Micro-Speakers. Audio Engineering Society, October 2012.

[11] Morten H. Knudsen and J. Grue Jensen. Low-Frequency Loudspeaker Models That Include Suspension Creep. *Journal of the Audio Engineering Society*, 41(1/2):3–18, February 1993.

[12] Neville Thiele. Loudspeakers in Vented Boxes: Part 1. *Journal of the Audio Engineering Society*, 19(5):382–392, May 1971.

[13] Werner Weselak and Gerhard Graber. *Skript zur Vorlesung "Elektroakustik"*. TU Graz.